AIMS Exercise Set # 3

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1. Which of the following matrices are regular? If reguolar, write down its LU factorization. (*a*) $\begin{pmatrix} 2 & 1 \\ 1 & 4 \end{pmatrix}$, (*b*) $\begin{pmatrix} 0 & -1 \\ 3 & -2 \end{pmatrix}$, (*c*) $\begin{pmatrix} 1 & -2 & 3 \\ -2 & 4 & -1 \\ 3 & -1 & 2 \end{pmatrix}$.

2. In each of the following problems, find the $A = LU$ factorization of the coefficient matrix, and then use Forward and Back Substitution to solve the corresponding linear systems $A\,\mathbf{x} = \mathbf{b}$ for each of the indicated right hand side:

(*a*) $A = \begin{pmatrix} -1 & 3 \\ 3 & 2 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$; (*b*) $A = \begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 2 & 3 & -1 \\ -1 & 3 & 2 & 2 \\ 0 & -1 & 2 & 1 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 1 \\ 0 \\ -1 \\ 1 \end{pmatrix}$.

3. Find the $L\,D\,L^T$ factorization of the matrix $\begin{pmatrix} 1 & -1 & -1 \\ -1 & 3 & 2 \\ -1 & 2 & 0 \end{pmatrix}$.

4. (*a*) Find the LU factorization of the $n \times n$ tridiagonal matrix A_n with all 2's along the diagonal and all -1's along the sub- and super-diagonals for $n = 3, 4$ and 5. (*b*) Use your factorizations to solve the system $A_n\,\mathbf{x} = \mathbf{b}$, where $\mathbf{b} = (1, 1, 1, \dots, 1)^T$. (*c*) Can you write down the LU factorization of A_n for general n? Do the entries in the factors approach a limit as n gets larger and larger?

5. *True or false*: (*a*) The product of two tridiagonal matrices is tridiagonal. (*b*) The inverse of a tridiagonal matrix is tridiagonal.

6. (*a*) Find the exact solution to the linear system $x - 5\,y - z = 1$, $\frac{1}{6}\,x - \frac{5}{6}\,y + z = 0$, $2\,x - y = 3$. (*b*) Solve the system using Gaussian Elimination with 4 digit rounding. (*c*) Solve the system using Partial Pivoting and 4 digit rounding. Compare your answers.

7. Implement the computer experiment with Hilbert matrices outlined in the last paragraph of the section.