Applied Linear Algebra

by Peter J. Olver and Chehrzad Shakiban

Corrections to Second Printing (2008)

Last updated: September 15, 2011

⋆⋆⋆ Page xvii ⋆⋆⋆

Add the following sentence at the end:

We would also like to thank Nihat Bayhan, Juan Cockburn, Richard Cook, Stephen DeSalvo, Anne Dougherty, Kathleen Fuller, Stuart Hastings, Roberta Jaskolski, Tian–Jun Li, James Meiss, Willard Miller, Jr., Timo Schürg, David Tieri, and Timothy Welle for sending us their comments, suggestions, and corrections to earlier printings of this book. A particular thanks to David Hiebeler for his careful reading and corrections.

⋆⋆⋆ Page 43 ⋆⋆⋆ Theorem 1.29:

To avoid confusion, change "having the nonzero pivots on the diagonal" to "with nonzero diagonal entries".

⋆⋆⋆ Page 43 ⋆⋆⋆ Theorem 1.31:

Insert "with nonzero diagonal entries" after "diagonal".

⋆⋆⋆ Page 51 ⋆⋆⋆ second displayed equation:

Replace the summand j by $j-1$:

$$\sum_{j=1}^{n} (j-1) = \frac{n^2-n}{2}$$

⋆⋆⋆ Page 57 ⋆⋆⋆ last displayed equation:

Replace 3210 by 32100:

$$10\,x + 1600\,y = 32100, \qquad x + .6\,y = 22,$$

⋆⋆⋆ Page 58 ⋆⋆⋆ first displayed equation:

Replace 3210 by 32100:

$$\left(\begin{array}{cc|c} 1600 & 10 & 32100 \\ .6 & 1 & 22 \end{array}\right)$$

$\star\star\star$ Page 100 $\star\star\star$ Exercise 2.3.39 (b):

Add closing bracket to $W[\,f(x), g(x)\,]$.

$\star\star\star$ Page 106 $\star\star\star$ Exercise 2.4.24 (b):

Change "Under the hypotheses of part (b)" to "Under the hypothesis of part (a)".

$\star\star\star$ Page 118 $\star\star\star$ lines 12–13:

Change "Solving the homogeneous system $\widehat{U}\mathbf{y} = \mathbf{0}$, we conclude that" to

The two nonzero rows of $\widehat{U}$ form a basis for corng A^T, and therefore

$\star\star\star$ Page 130 $\star\star\star$ line 6 and line 8:

Delete first "both"'s:

Inner products and norms lie at the heart of linear (and nonlinear) analysis, in both finite-dimensional vector spaces and infinite-dimensional function spaces. It is impossible to overemphasize their importance for theoretical developments, practical applications, and the design of numerical solution algorithms.

$\star\star\star$ Page 143 $\star\star\star$ Exercise 3.2.31 (a):

Add ^{T} superscripts to $(\,1, 2, 3\,)^T$ and $(\,1, -1, 2\,)^T$.

$\star\star\star$ Page 161 $\star\star\star$ Formula before Proposition 3.34:

Change dt to dx.

$\star\star\star$ Page 161 $\star\star\star$ Two lines before Proposition 3.34:

Change Theorem 3.31 to Theorem 3.28.

$\star\star\star$ Page 162 $\star\star\star$ Exercise 3.4.22 (c):

Change "null vectors" to "null directions".

$\star\star\star$ Page 162 $\star\star\star$ Exercise 3.4.32:

Change "null vector" to "null direction" and $K = A^T A$ to $K = A^T C\,A$:

Show that $\mathbf{0} \neq \mathbf{z}$ is a null direction for the quadratic form $q(\mathbf{x}) = \mathbf{x}^T K\,\mathbf{x}$ based on the Gram matrix $K = A^T C\,A$ if and only if $\mathbf{z} \in \ker K$.

$\star\star\star$ Page 163 $\star\star\star$ Exercise 3.4.35 (d):

Rephrase for clarity:

Show that K is also a Gram matrix, by finding a matrix A such that $K = A^T C\,A$.

⋆⋆⋆ Page 168 ⋆⋆⋆ Sentence after that containing (3.70):

Rephrase for clarity:

Note that M is a lower triangular matrix with all positive diagonal entries, namely the square roots of the pivots: $m_{ii} = \sqrt{d_i}$.

⋆⋆⋆ Page 177 ⋆⋆⋆ Exercise 3.6.29:

Delete part (e). ("Orthogonal" and "orthonormal" are not yet defined.)

⋆⋆⋆ Page 179 ⋆⋆⋆ Exercise 3.6.51:

For each of the following . . .

⋆⋆⋆ Page 188 ⋆⋆⋆ Formula in middle of page:

y^* and z^* should not be bold face.

⋆⋆⋆ Page 188 ⋆⋆⋆ Theorem 4.4:

Delete the words "null vector".

⋆⋆⋆ Page 191 ⋆⋆⋆ Equation (4.26):

The last equality, $c = \| \mathbf{b} \|^2$, is correct provided one uses the weighted norm. However, to avoid confusion with the Euclidean norm used in (4.25), it would be better to write this as $c = \mathbf{b}^T C \mathbf{b}$.

⋆⋆⋆ Page 194 ⋆⋆⋆ Equation (4.30):

$$\| A\mathbf{x}^\star - \mathbf{b} \| = \sqrt{\| \mathbf{b} \|^2 - \mathbf{f}^T \mathbf{x}^\star} = \sqrt{\| \mathbf{b} \|^2 - \mathbf{b}^T A (A^T A)^{-1} A^T \mathbf{b}} \, .$$

⋆⋆⋆ Page 201 ⋆⋆⋆ line after Equation (4.43) :

Delete first "coefficient":

The $m \times (n+1)$ coefficient matrix . . .

⋆⋆⋆ Page 221 ⋆⋆⋆ Exercise 5.1.11:

$$\mathbf{u}_2 = \pm \begin{pmatrix} -\sin\theta \\ \cos\theta \end{pmatrix}$$

⋆⋆⋆ Page 231 ⋆⋆⋆ Second line:

Replace "For exercises #1–8 use " by "For Exercises #1–7 use ".

⋆⋆⋆ Page 245 ⋆⋆⋆ First line under Figure:

Change "**x** and **y**" to "**v** and **w**".

⋆⋆⋆ Page 274 ⋆⋆⋆ Remark:

. . . solving the homogeneous adjoint system, . . .

⋆⋆⋆ Page 276 ⋆⋆⋆ Exercise 5.6.20 (d):

Change the sign in front of $4x_3$ in last equation:

$$x_1 + 2x_2 + 3x_3 = b_1, \quad x_2 + 2x_3 = b_2, \quad 3x_1 + 5x_2 + 7x_3 = b_3, \quad -2x_1 + x_2 + 4x_3 = b_4;$$

⋆⋆⋆ Page 279 ⋆⋆⋆ Equation (5.90):

Change $e^{\mathrm{i}\,k\,x_n}$ to $e^{\mathrm{i}\,k\,x_{n-1}}$ in first line.

⋆⋆⋆ Page 285 ⋆⋆⋆ Line -5:

Change $n = 2^8 = 256$ to $n = 2^9 = 512$.

⋆⋆⋆ Page 296 ⋆⋆⋆ Equation (6.9):

Insert space between 1 and -1 in last row of matrix.

⋆⋆⋆ Page 298 ⋆⋆⋆ Two lines before (6.15):

Change $K\mathbf{x} = \mathbf{f}$ to $K\mathbf{u} = \mathbf{f}$.

⋆⋆⋆ Page 298 ⋆⋆⋆ Four lines after (6.15):

Change $\mathbf{y} = A^{-1}\mathbf{f}$ to $\mathbf{y} = A^{-T}\mathbf{f}$.

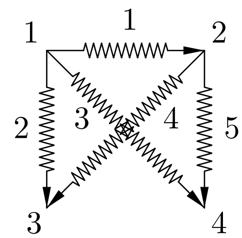
⋆⋆⋆ Page 311 ⋆⋆⋆ Exercise 6.2.1 (b):

Change last row of matrix:

$$\begin{pmatrix} 0 & 0 & 1 & -1 \\ 1 & 0 & 0 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & 0 & -1 & 0 \end{pmatrix}$$

⋆⋆⋆ Page 311 ⋆⋆⋆ Exercise 6.2.2:

Add labels to the wires:



⋆⋆⋆ Page 312 ⋆⋆⋆ Exercise 6.2.12 (a):

Start the exercise with:

Assuming all wires have unit resistance, find the voltage . . .

⋆⋆⋆ Page 313 ⋆⋆⋆ Line 19:

Delete “the” before “Section 6.1”.

⋆⋆⋆ Page 317 ⋆⋆⋆ Displayed equation above (6.51):

Change 0 to **0**.

⋆⋆⋆ Page 319 ⋆⋆⋆ Equation (6.58):

Change the last formula to

$$\mathbf{z}_3 \cdot \mathbf{f} = -\frac{\sqrt{3}}{2}\, f_1 + \frac{1}{2}\, g_1 + g_2 = 0.$$

⋆⋆⋆ Page 319 ⋆⋆⋆ Last line:

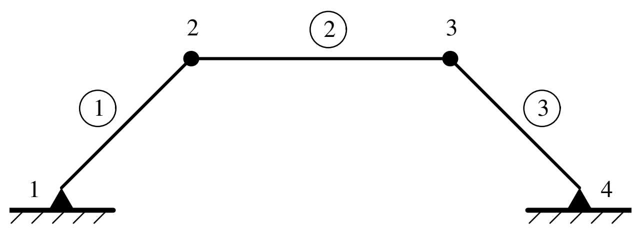
Change "first node" to "third node".

⋆⋆⋆ Page 320 ⋆⋆⋆ Two lines before displayed equation for $A^{\star\star}$:

This serves to also eliminate ...

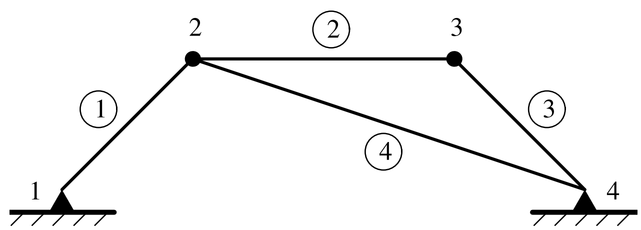
⋆⋆⋆ Page 321 ⋆⋆⋆ Figure 6.13:

Label the bars in the figure:



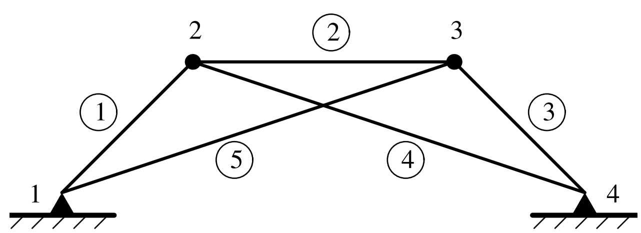
⋆⋆⋆ Page 323 ⋆⋆⋆ Figure 6.16:

Label the bars in the figure:



⋆⋆⋆ Page 324 ⋆⋆⋆ Figure 6.17:

Label the bars in the figure:



$\star\star\star$ Page 325 $\star\star\star$ Line -5:

Change "three bars" to "five bars".

$\star\star\star$ Page 339 $\star\star\star$ Equation (7.12):

Add period at end of equation.

$\star\star\star$ Page 384 $\star\star\star$ Exercise 7.5.1 (b):

$\langle\, \mathbf{v}\,;\mathbf{w}\,\rangle = 2\,v_1\,w_1 + 3\,v_2\,w_2$

$\star\star\star$ Page 399 $\star\star\star$ Final displayed equation:

The bar on the second term should extend over both A and $\mathbf{v}$:

$$\overline{A}\,\overline{v} = \overline{A\,\mathbf{v}} = \overline{\lambda\,\mathbf{v}} = \overline{\lambda}\,\overline{v}.$$

$\star\star\star$ Page 400 $\star\star\star$ Two lines before Remark:

Change "combinations of the real eigenvalues" to " combinations of the real eigenvectors".

$\star\star\star$ Page 411 $\star\star\star$ Last line:

Delete "the" before "Section 8.6".

$\star\star\star$ Page 432 $\star\star\star$ Fourth displayed formula. Switch ^{T} superscript:

$$A^+ = Q\,\Sigma^{-1} P^T = \begin{pmatrix} .2444 & .1333 & .0556 & .1889 \\ .1556 & -.0667 & .1111 & .0444 \\ -.1111 & 0 & -.0556 & -.0556 \end{pmatrix}.$$

$\star\star\star$ Page 572 $\star\star\star$ Displayed formula before (10.102):

The subscripts on R and Q are wrong:

$$A_2 = R_1\,Q_1.$$

$\star\star\star$ Page 575 $\star\star\star$ Change equations (10.106) and (10.107) to:

$$A^k = \big(\,Q_0\,Q_1 \cdots Q_{k-1}\,\big)\,\big(\,R_{k-1}\cdots R_1\,R_0\,\big). \tag{10.106}$$

$$\begin{aligned} S_k &= Q_0\,Q_1 \cdots Q_{k-1} = S_{k-1}\,Q_{k-1}, \\ P_k &= R_{k-1}\cdots R_2\,R_1 = R_{k-1}\,P_{k-1}. \end{aligned} \tag{10.107}$$

⋆⋆⋆ Page 577 ⋆⋆⋆ Replace the paragraph after Theorem 10.57 by the following::

The last remaining item is a proof of Lemma 10.56. We write

$$S = (\,\mathbf{u}_1\ \mathbf{u}_2\ \ldots\ \mathbf{u}_n\,), \qquad S_k = (\mathbf{u}_1^{(k)}, \ldots, \mathbf{u}_n^{(k)})$$

in columnar form. Let $t_{ij}^{(k)}$ denote the entries of the positive upper triangular matrix T_k. The first column of the limiting equation $S_k\,T_k \to S$ reads

$$t_{11}^{(k)}\,\mathbf{u}_1^{(k)} \;\longrightarrow\; \mathbf{u}_1.$$

Since both $\mathbf{u}_1^{(k)}$ and $\mathbf{u}_1$ are unit vectors, and $t_{11}^{(k)} > 0$,

$$\|\,t_{11}^{(k)}\,\mathbf{u}_1^{(k)}\,\| = t_{11}^{(k)} \;\longrightarrow\; \|\,\mathbf{u}_1\,\| = 1, \quad \text{and hence} \quad \mathbf{u}_1^{(k)} \;\longrightarrow\; \mathbf{u}_1.$$

The second column reads

$$t_{12}^{(k)}\,\mathbf{u}_1^{(k)} + t_{22}^{(k)}\,\mathbf{u}_2^{(k)} \;\longrightarrow\; \mathbf{u}_2.$$

Taking the inner product with $\mathbf{u}_1^{(k)} \to \mathbf{u}_1$ and using orthonormality, we deduce $t_{12}^{(k)} \to 0$, and so $t_{22}^{(k)}\,\mathbf{u}_2^{(k)} \to \mathbf{u}_2$, which, by the previous reasoning, implies $t_{22}^{(k)} \to 1$ and $\mathbf{u}_2^{(k)} \to \mathbf{u}_2$. The proof is completed by working in order through the remaining columns, employing a similar argument at each step. Details are left to the interested reader.

⋆⋆⋆ Page 591 ⋆⋆⋆ Equation (11.21):

Insert minus sign before integral:

$$u'(\ell) = -\int_0^\ell f(x)\,dx = 0, \tag{11.21}$$

⋆⋆⋆ Page 594 ⋆⋆⋆ Line before (11.28):

Change “to satisfy” to “satisfy”.

⋆⋆⋆ Page 598 ⋆⋆⋆ 3 lines after (11.40):

Change $L[u] = u(y)$ to $L_y[u] = u(y)$.

⋆⋆⋆ Page 607 ⋆⋆⋆ equation (11.60):

Missing factor of c in differential equation:

$$-c\,u'' = f(x), \qquad\qquad u(0) = 0 = u(1),$$

⋆ ⋆ ⋆ Page 608 ⋆ ⋆ ⋆ Lines 10 and 7 from bottom:

Two missing factors of c:

$$c\,\frac{du}{dx} = (1-x)\,x\,f(x) + \int_0^x \left[-y\,f(y)\right] dy - x\,(1-x)\,f(x) + \int_x^1 (1-y)\,f(y)\,dy$$
$$= -\int_0^1 y\,f(y)\,dy + \int_x^1 f(y)\,dy.$$

Differentiating again, we conclude that $-c\,\dfrac{d^2u}{dx^2} = f(x)$, as claimed.

⋆ ⋆ ⋆ Page 619 ⋆ ⋆ ⋆ Exercise 11.3.16 (b):

Delete "is" after $K = L^* \circ L$.

⋆ ⋆ ⋆ Page 653 ⋆ ⋆ ⋆ Solution 1.2.4 (d):

$$A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & -1 & 3 \\ 3 & 0 & -2 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} u \\ v \\ w \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix};$$

⋆ ⋆ ⋆ Page 653 ⋆ ⋆ ⋆ Solution 1.2.4 (f):

$$\mathbf{b} = \begin{pmatrix} -3 \\ -5 \\ 2 \\ 1 \end{pmatrix}.$$

⋆ ⋆ ⋆ Page 655 ⋆ ⋆ ⋆ Solution 1.4.15 (a):

$$\begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix}.$$

⋆ ⋆ ⋆ Page 658 ⋆ ⋆ ⋆ Solution 1.8.4:

(i) $a \neq b$ and $b \neq 0$; (ii) $a = b \neq 0$, or $a = -2,\ b = 0$; (iii) $a \neq -2,\ b = 0$.

⋆ ⋆ ⋆ Page 659 ⋆ ⋆ ⋆ Solution 1.8.23 (e):

$(\,0, 0, 0\,)^T$;

⋆ ⋆ ⋆ Page 665 ⋆ ⋆ ⋆ Solution 3.4.22 (v):

Change "null vectors" to "null directions".

⋆ ⋆ ⋆ Page 665 ⋆ ⋆ ⋆ Solution 3.4.32:

Change all $\mathbf{x}$'s to $\mathbf{z}$:

$0 = \mathbf{z}^T K\,\mathbf{z} = \mathbf{z}^T A^T C\,A\,\mathbf{z} = \mathbf{y}^T C\,\mathbf{y}$, where $\mathbf{y} = A\,\mathbf{z}$. Since $C > 0$, this implies $\mathbf{y} = \mathbf{0}$, and hence $\mathbf{z} \in \ker A = \ker K$.

⋆⋆⋆ Page 667 ⋆⋆⋆ Solution 4.4.27 (a):

Change "the interpolating polynomial" to "an interpolating polynomial".

⋆⋆⋆ Page 667 ⋆⋆⋆ Solution 4.4.52 (b):

Delete the sentence:

(The solution given in the text is for the square $S = \left\{ 0 \le x \le 1,\ 0 \le y \le 1 \right\}$.)

⋆⋆⋆ Page 668 ⋆⋆⋆ Solution 5.1.14 (b):

$$\mathbf{v}_2 = \pm \left(-\sin\theta, \tfrac{1}{\sqrt{2}} \cos\theta \right)^T$$

⋆⋆⋆ Page 670 ⋆⋆⋆ Solution 5.4.15:

$$p_0(x) = 1, \qquad p_1(x) = x, \qquad p_2(x) = x^2 - \tfrac{1}{3}, \qquad p_3(x) = x^3 - \tfrac{9}{10}\, x.$$

(The solution given in the text is for the interval $[0, 1]$, not $[-1, 1]$.)

⋆⋆⋆ Page 670 ⋆⋆⋆ Solution 5.5.6 (ii) (c):

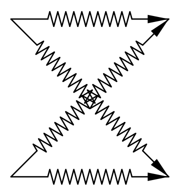
$$\left(\tfrac{23}{43}, \tfrac{19}{43}, -\tfrac{1}{43} \right)^T.$$

⋆⋆⋆ Page 674 ⋆⋆⋆

The page layout is a bit strange. The top of the second column (before Solution 6.2.1) is the solution to Exercise 6.1.16(c). Also, the solution to Exercise 6.2.10 spans across both columns.

⋆⋆⋆ Page 674 ⋆⋆⋆ Solution 6.2.1 (b):

The solution corresponds to the revised exercise — see correction on page 311.

For the given matrix, the solution is 

⋆⋆⋆ Page 674 ⋆⋆⋆ Solution 6.2.12 & 6.2.13:

Change all $\mathbf{e}$'s to $\mathbf{y}$'s.

⋆⋆⋆ Page 674 ⋆⋆⋆ Solution 6.3.5 (b):

$$\begin{aligned} \tfrac{3}{2}\, u_1 - \tfrac{1}{2}\, v_1 - u_2 &= f_1, \\ -\tfrac{1}{2}\, u_1 + \tfrac{3}{2}\, v_1 &= g_1, \\ -u_1 + \tfrac{3}{2} u_2 + \tfrac{1}{2}\, v_2 &= f_2, \\ \tfrac{1}{2} u_2 + \tfrac{3}{2}\, v_2 &= g_2. \end{aligned}$$

$\star\star\star$ Page 684 $\star\star\star$ Solution 8.5.26:

Change (b) to (c).

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.2.8 (d):

$$\begin{aligned} f'(x) &= 4\,\delta(x+2) + 4\,\delta(x-2) + \begin{cases} 1, & |\,x\,| > 2, \\ -1, & |\,x\,| < 2, \end{cases} \\ &= 4\,\delta(x+2) + 4\,\delta(x-2) + 1 - 2\,\sigma(x+2) + 2\,\sigma(x-2), \\ f''(x) &= 4\,\delta'(x+2) + 4\,\delta'(x-2) - 2\,\delta(x+2) + 2\,\delta(x-2). \end{aligned}$$

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.2.31 (a):

$$u_n(x) = \begin{cases} x\,(1-y), & 0 \le x \le y - \frac{1}{n}, \\ -\frac14 n\,x^2 + \left(\frac12 n - 1\right) x\,y - \frac14 n\,y^2 + \frac12 y + \frac12 x - \frac{1}{4n}, & |\,x - y\,| \le \frac{1}{n}, \\ y\,(1-x), & y + \frac{1}{n} \le x \le 1. \end{cases}$$

$\star\star\star$ Page 694 $\star\star\star$ Solution 11.3.3 (c):

(*i*) $u_\star(x) = \frac12 x^2 - \frac52 + x^{-1}$,

(*ii*) $\mathcal{P}[u] = \displaystyle\int_1^2 \left[\, \tfrac12 x^2 (u')^2 + 3x^2 u \,\right] dx, \qquad u'(1) = u(2) = 0,$

(*iii*) $\mathcal{P}[u_\star] = -\frac{37}{20} = -1.85$,

(*iv*) $\mathcal{P}[x^2 - 2x] = -\frac{11}{6} = -1.83333, \quad \mathcal{P}\left[-\sin\frac12 \pi x\right] = -1.84534.$

$\star\star\star$ Page 696 $\star\star\star$ Solution 11.5.7 (b):

$$\lambda = -\omega^2 < 0, \qquad G(x,y) = \begin{cases} \dfrac{\sinh\omega\,(y-1)\,\sinh\omega\,x}{\omega\sinh\omega}, & x < y, \\ \dfrac{\sinh\omega\,(x-1)\sinh\omega\,y}{\omega\sinh\omega}, & x > y; \end{cases}$$

$$\lambda = 0, \qquad G(x,y) = \begin{cases} x(y-1), & x < y, \\ y(x-1), & x > y; \end{cases}$$

$$\lambda = \omega^2 \ne n^2\pi^2 > 0, \qquad G(x,y) = \begin{cases} \dfrac{\sin\omega\,(y-1)\,\sin\omega\,x}{\omega\sin\omega}, & x < y, \\ \dfrac{\sin\omega\,(x-1)\sin\omega\,y}{\omega\sin\omega}, & x > y. \end{cases}$$

