

CORRECTIONS TO FIRST AND SECOND PRINTINGS OF

Olver, P.J., *Equivalence, Invariants, and Symmetry*,

Cambridge University Press, Cambridge, 1995.

Last modified: October 2, 2009

*** *On back cover, line 17–18, change*

prospective geometry

to

projective geometry

*** *page xv, add to acknowledgements*

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*** *page 22, line 10, change*

and all $t, s \in \mathbb{R}$ where the equation is defined.

to

all $t, s \in V$ where $V \subset \mathbb{R}^2$ is a connected open subset of the (t, s) plane containing $(0, 0)$ consisting of points where the equation is defined.

*** *page 39, Example 2.13, change the first two occurrences of*

$\mathrm{PSL}(n, \mathbb{R})$

to

$\mathrm{PGL}(n, \mathbb{R})$.

*** *Also append to the last sentence*

$\mathrm{PSL}(n, \mathbb{R}) = \mathrm{SL}(n, \mathbb{R})/\{\pm \mathbb{1}\}$ is equal to the connected component of $\mathrm{PGL}(n, \mathbb{R})$ containing the identity.

*** *page 51, equation (2.14), change*

$$C_{ij}^k = -C_{ij}^k$$

to

$$C_{ji}^k = -C_{ij}^k$$

*** *page 55, lines 4–5, change*

$G_H = \{g|gHg^{-1} \subset H\}$ has Lie algebra

to

$G_H = \{g|gHg^{-1} \subset H\}$ is a normal subgroup with Lie algebra

*** *page 61, line 31, change*

there is a scalar function $h_{\mathbf{v}}(t)$ such that

to

there is a function $h_{\mathbf{v}}: \mathbb{R}^k \rightarrow \mathbb{R}^k$ such that

*** page 73, line 9, change

$$\begin{pmatrix} a^{-1} da & a^{-1}(a db - b da) \\ 0 & 1 \end{pmatrix}$$

to

$$\begin{pmatrix} a^{-1} da & a^{-1} db \\ 0 & 0 \end{pmatrix}$$

*** page 85, equation (3.18), change

$$1 + th_{\mathbf{v}}(x) + \frac{1}{2}t^2\mathbf{v}(h_{\mathbf{v}}) + \dots$$

to

$$1 + th_{\mathbf{v}}(x) + \frac{1}{2}t^2[\mathbf{v}(h_{\mathbf{v}}) + h_{\mathbf{v}}^2] + \dots$$

*** page 87, equation (3.21), change

$$\sigma([\mathbf{v}, \mathbf{w}]) = \widehat{\mathbf{w}}(\sigma(\mathbf{v})) - \widehat{\mathbf{v}}(\sigma(\mathbf{w}))$$

to

$$\sigma([\mathbf{v}, \mathbf{w}]) = \widehat{\mathbf{v}}(\sigma(\mathbf{w})) - \widehat{\mathbf{w}}(\sigma(\mathbf{v}))$$

*** page 93, lines 10–24, change

In order to formulate a general theorem governing the existence of relative invariants for sufficiently regular group actions, we consider the extended group action (3.15) on the bundle $E = M \times U$. The key remark is that there is a one-to-one correspondence between relative invariants of weight μ and linear absolute invariants of the extended action. Specifically, a linear function $J(x, u) = \sum_{\alpha=1}^n R_{\alpha}(x)u^{\alpha}$ is an invariant of the extended group action (3.15) if and only if the vector-valued function $R(x) = (R_1(x), \dots, R_q(x))^T$ is a relative invariant of weight μ . Therefore, we need only produce a sufficient number of *linear* invariants of the extended action. Moreover, if $J(x, u)$ is any invariant of the extended group action, then it is not hard to prove that its linear Taylor polynomial is also an invariant, and hence provides a relative invariant for the multiplier representation. Thus, the only question is how many independent relative invariants can be constructed in this manner.

to

In order to formulate a general theorem governing the existence of relative invariants for sufficiently regular group actions, we consider the extended group action (3.15) on the bundle $E = M \times U$ and its dual version $(x, v) \mapsto (g \cdot x, \mu(g, x)^{-T})$ on the dual bundle $E^* = X \times U^*$. The key remark is that there is a one-to-one correspondence between relative invariants of weight μ and linear absolute invariants of the dual action. Specifically, a linear function $J(x, v) = \sum_{\alpha=1}^n R_{\alpha}(x)v^{\alpha}$ is an invariant of the dual action on E^* if and only if the vector-valued function $R(x) = (R_1(x), \dots, R_q(x))^T$ is a relative invariant of weight μ . Therefore, we need only produce a sufficient number of *linear* invariants of the extended action. Moreover, if $J(x, v)$ is any invariant of the extended group action, then it is not hard to prove that its linear Taylor polynomial is also an invariant, and hence provides a relative invariant for the multiplier representation. Thus, the only question is how many independent relative invariants can be constructed in this manner.

*** page 94, lines 26–28, change

I do not know a general theorem that counts the number of relative invariants of multiplier representations that do not satisfy the hypotheses of Theorem 3.36

to

A general theorem that counts the number of relative invariants of multiplier representations in all cases can be found in the recent paper by M. Fels and the author, “On relative invariants”, *Math. Ann.* **308** (1997), 701–732.

*** page 96, equation (3.30), change

$$\begin{aligned}\mathbf{v}_- &= a_1 \frac{\partial}{\partial a_0} + 2a_2 \frac{\partial}{\partial a_1} + \cdots + (n-1)a_{n-1} \frac{\partial}{\partial a_{n-2}} + na_n \frac{\partial}{\partial a_{n-1}}, \\ \mathbf{v}_0 &= -na_0 \frac{\partial}{\partial a_0} - (n-2)a_1 \frac{\partial}{\partial a_1} + \cdots + (n-2)a_{n-1} \frac{\partial}{\partial a_{n-1}} + na_n \frac{\partial}{\partial a_n}, \\ \mathbf{v}_+ &= na_0 \frac{\partial}{\partial a_1} + (n-1)a_1 \frac{\partial}{\partial a_2} + \cdots + 2a_{n-2} \frac{\partial}{\partial a_{n-1}} + a_{n-1} \frac{\partial}{\partial a_n}.\end{aligned}$$

to

$$\begin{aligned}\mathbf{v}_- &= na_1 \frac{\partial}{\partial a_0} + (n-1)a_2 \frac{\partial}{\partial a_1} + \cdots + 2a_{n-1} \frac{\partial}{\partial a_{n-2}} + a_n \frac{\partial}{\partial a_{n-1}}, \\ \mathbf{v}_0 &= na_0 \frac{\partial}{\partial a_0} + (n-2)a_1 \frac{\partial}{\partial a_1} + \cdots + (2-n)a_{n-1} \frac{\partial}{\partial a_{n-1}} - na_n \frac{\partial}{\partial a_n}, \\ \mathbf{v}_+ &= a_0 \frac{\partial}{\partial a_1} + 2a_1 \frac{\partial}{\partial a_2} + \cdots + (n-1)a_{n-2} \frac{\partial}{\partial a_{n-1}} + na_{n-1} \frac{\partial}{\partial a_n}.\end{aligned}$$

*** page 108, line 24, change

$$\cot \theta \neq a$$

to

$$\cot t \neq a$$

*** page 120, second line after equation (4.35), change

The Lie algebra (4.14)

to

The Lie algebra (4.35)

*** page 126, line 12, change

$$(\Psi^{(n)})^* \theta$$

to

$$\Psi^* \theta$$

*** page 142, line 28, change

$$s_0 = 1, s_1 = 2, \dots, s_{r-3} = s_{r-2} = r - 1$$

to

$$s_0 = 2, s_1 = 3, \dots, s_{r-3} = s_{r-2} = r - 1$$

*** page 144, line 10, change

$$a_\mu^\nu \xi_\nu^i$$

to

$$A_\mu^\nu \xi_\nu^i$$

*** page 148, equation (5.15), change

$$\mathbf{v}_0 = x \frac{\partial}{\partial x} - \frac{n}{2} u \frac{\partial}{\partial u}, \quad \mathbf{v}_+ = x^2 \frac{\partial}{\partial x} - nxu \frac{\partial}{\partial u}.$$

to

$$\mathbf{v}_0 = x \frac{\partial}{\partial x} + \frac{n}{2} u \frac{\partial}{\partial u}, \quad \mathbf{v}_+ = x^2 \frac{\partial}{\partial x} + nxu \frac{\partial}{\partial u}.$$

*** page 159, lines 5, 15 & 18, change

$$d_{n+1}K_1 \wedge \dots \wedge d_{n+1}K_r$$

to

$$d_{n+1}[\mathcal{D}K_1] \wedge \dots \wedge d_{n+1}[\mathcal{D}K_r]$$

*** page 171, lines 20 & -8, change

$$n + 2$$

to

$$n + 1$$

*** page 171, line -7 to -3, delete sentence

Moreover, if the stable ... have order at most $n + 1$.

*** page 173, Example 5.52, line 2, after "... via the standard representation", add

$$(x, y, u) \mapsto (\alpha x + \beta y, \gamma x + \delta y, u), \text{ where } \alpha\delta - \beta\gamma = 1$$

*** page 188, line -2, change

$$\log x = h(u/x)$$

to

$$\log x = h(u/x^m)$$

*** page 190, lines 8–9, change

H -reduced equationsymmetry reduced equation $\Delta/H = 0$ admits the corresponding normalizer subgroup $G_H = \{g|g \cdot H \cdot g^{-1} \subset H\}$ as a symmetry group.

to

H -reduced equation $\Delta/H = 0$ admits the quotient group G_H/H , where $G_H = \{g|g \cdot H \cdot g^{-1} \subset H\}$ is the normalizer subgroup, as a symmetry group.

*** page 190, line 18, change

$$\eta\partial_y + \zeta\partial_u + \zeta^y\partial_{v_y}$$

to

$$\eta\partial_y + \zeta\partial_v + \zeta^y\partial_{v_y}$$

*** page 190, line 22, change

$$\mathbf{v} = \partial_y$$

to

$$\mathbf{v} = \partial_v$$

*** page 192, formula (6.32), change

$$(1 + u_x)^{3/2}$$

to

$$(1 + u_x^2)^{3/2}$$

*** page 192, displayed formula after (6.32), change

$$(1 + \theta_r^2)$$

to

$$(1 + r^2\theta_r^2)^{3/2}$$

*** page 195, line -4, change

Alternatively, $x = w_{uu}/w_u$, where w is an arbitrary solution ...

to

Alternatively, $w = x_{uu}/x_u$ is an arbitrary solution ...

*** page 198, line 9, change

$$y = f(x)$$

to

$$w = f(x)$$

*** page 198, equation (6.56), change

$$y$$

to

$$w$$

*** page 201, equation (6.61), change

$$\det \begin{vmatrix} \xi_1 & \varphi_1 & \varphi_1^1 & \cdots & \varphi_1^{r-1} \\ \xi_2 & \varphi_2 & \varphi_2^1 & \cdots & \varphi_2^{r-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \xi_r & \varphi_r & \varphi_r^1 & \cdots & \varphi_r^{r-1} \end{vmatrix} = 0.$$

to

$$\det \begin{vmatrix} \xi_1 & \varphi_1 & \varphi_1^1 & \cdots & \varphi_1^{r-2} \\ \xi_2 & \varphi_2 & \varphi_2^1 & \cdots & \varphi_2^{r-2} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \xi_r & \varphi_r & \varphi_r^1 & \cdots & \varphi_r^{r-2} \end{vmatrix} = 0.$$

*** page 226, line 6, change

$$P(t, x, u^{(2n)})$$

to

$$R(t, x, u^{(2n)})$$

*** page 231, lines -4 & -1, change

$$E(\bar{L})$$

to

$$\bar{E}(\bar{L})$$

*** page 243, lines 18 & 20, change

$$(x, v_y, v_{yy}, \dots)$$

to

$$(y, v_y, v_{yy}, \dots)$$

*** page 293, line 7, change

$$a_4 = 0$$

to

$$a_4 = a_5 = 0$$

*** page 293, equations (9.30) & (9.32), change

$$\bar{a}_6 \omega^3 = a_6 \omega^3$$

to

$$\bar{a}_6 \bar{\omega}^3 = a_6 \omega^3$$

*** page 307, line 13, change

$$\tilde{\alpha}^\kappa = \sum_k z_j^\kappa(x) \theta^j$$

to

$$\tilde{\alpha}^\kappa = \sum_j z_j^\kappa(x) \theta^j$$

*** page 307, equation (10.7), change

$$\sum_{k=1}^r z_j^\kappa \theta^j$$

to

$$\sum_{j=1}^m z_j^\kappa \theta^j$$

*** page 309, equation (10.12), change

$$\sum_{i=1}^p z_i^\kappa \theta^i$$

to

$$\sum_{i=1}^m z_i^\kappa \theta^i$$

*** page 339, line 6, delete first

arc length

*** page 341, line -3, change

$$I_4$$

to

$$I_5$$

*** page 349, line -12, change

$$\alpha^1 - T_{12}^1 \theta^1 \wedge \theta^2 - T_{13}^1 \theta^1 \wedge \theta^3$$

to

$$\alpha^1 - T_{12}^1 \theta^2 - T_{13}^1 \theta^3$$

*** page 367, line 10, change

manifolds M

to

manifolds M and \overline{M}

*** page 368, equation (11.30), change

$$= T \omega^1 \wedge \omega^2 \wedge \omega^3 = T\Omega.$$

to

$$= T \omega^1 \wedge \omega^2 \wedge \omega^3.$$

*** page 372, lines 13–16, change

However, I do not know any naturally occurring examples exhibiting this phenomenon, and, moreover, the prolongation procedure to be discussed below will handle this (remote) possibility as well.)

to

However, the prolongation procedure to be discussed below will handle this possibility as well; an example is the equivalence problem for a parabolic evolution equation analyzed in [69].)

*** page 375, line 5, change

$$(12.3)$$

to

$$(12.1)$$

*** page 394, lines 16 & 21, change

$$(11.6)$$

to

$$(11.7)$$

*** page 394, line 22, change

vector S

to

matrix S

*** page 395, equation (12.52), change

$$\varpi = \alpha + S \theta, \quad \text{or explicitly,} \quad \varpi^i = \alpha^i + \sum_{j=1}^m S_j^i \theta^j$$

to

$$\varpi = \alpha - S \theta, \quad \text{or explicitly,} \quad \varpi^i = \alpha^i - \sum_{j=1}^m S_j^i \theta^j$$

*** page 406, equation (12.73), change

$$Q_p \widehat{D}_x Q_{pp} 6 Q_{uu}$$

to

$$Q_p \widehat{D}_x Q_{pp} + 6 Q_{uu}$$

*** page 411, lines 12–13, change

$$c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial x} = a(x, y, \varphi(x, y)),$$
$$c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial y} = b(x, y, \varphi(x, y)).$$

to

$$c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial x} = -a(x, y, \varphi(x, y)),$$
$$c(x, y, \varphi(x, y)) \frac{\partial \varphi}{\partial y} = -b(x, y, \varphi(x, y)).$$

*** page 423, equation (14.4), change

$$\Phi(t, w)$$

to

$$\Phi(t, s)$$

*** page 425, lines 3–6, change

There is, however, a four-parameter group action obtained by including the additional generator $z\partial_y$, whose associated one-parameter group $(x, y, z) \mapsto (x, y + \mu z, z)$ can be recovered from the previous group transformations by taking commutators.

to

Moreover, one cannot include these vector fields in a finite-dimensional Lie algebra, since $[\mathbf{v}_2, \mathbf{v}_3] = \mathbf{v}_4 = z\partial_y$, $[\mathbf{v}_4, \mathbf{v}_3] = \mathbf{v}_5 = z^2\partial_y$, and so on, hence the successive commutators span an infinite-dimensional Lie algebra of vector fields.

*** pages 425, lines 33–34, change

Relative invariants correspond to *linear* invariants $J(x, u) = R(x) \cdot u = \sum_{\alpha=1}^q R_\alpha(x) u^\alpha$ of the extended action, ...

to

Relative invariants of the dual action on $E^* = X \times U^*$ correspond to *linear* invariants $J(x, u) = \sum_{\alpha=1}^q R_\alpha(x) u^\alpha$ of the extended action, ...

*** page 472, Table 1, Case 1.8, column 3, change

$$\mathfrak{a}(1) \times \mathbb{C}^k$$

to

$$\mathbb{C} \times (\mathbb{C} \times \mathbb{C}^k)$$

*** pages 477, 478, 480, 484, 486 & 487, update the following references:

[8] Anderson, I.M., and Kamran, N., The variational bicomplex for second order scalar partial differential equations in the plane, *Duke Math. J.* **87** (1997), 265–319

[29] Bryant, R.L., and Griffiths, P.A., Characteristic cohomology of differential systems I, II, *J. Amer. Math. Soc.* **8** (1995), 507–596, *Duke Math. J.* **78** (1995), 531–676.

[30] Bryant, R.L., Griffiths, P.A., and Hsu, L.; Hyperbolic exterior differential systems and their conservation laws, Part I, *Selecta Math.* **1** (1995), 21–112.

[70] Fels, M., The equivalence problem for systems of second-order ordinary differential equations, *Proc. London Math. Soc.* **71** (1995), 221–240

[139] Komrakov, B., Primitive actions and the Sophus Lie problem, in: *The Sophus Lie Memorial Conference, Oslo, 1992*, O.A. Laudal and B. Jahren, eds., Scandinavian Univ. Press, Oslo, 1994, pp. 187–269

[188] Olver, P.J., Non-associative local Lie groups, *J. Lie Theory* **6** (1996), 23–51.

[190] Olver, P.J., Sapiro, G., and Tannenbaum, A., Invariant geometric evolutions of surfaces and volumetric smoothing, *SIAM J. Appl. Math.* **57** (1997), 176–194.

*** page 479, refs [37–38], change

Complètes

to

Complètes

*** page 479, refs [37–42], change

Gauthiers

to

Gauthier

*** page 483, reference [128], change

dx/dy

to

dy/dx

*** page 500, change

Galois, E., 3

to

Galois, E., 4

*** page 501, change

Morikawa, H., 217, [170–172]

to

Morikawa, H., 217, [170]

Morrey, C.B., Jr., 346, [171]

Mostow, G.D., 41, 61, [172]

*** page 504, change two entries

affine-invariant arc length, 339

to

affine-invariant arc length, 241, 339