Corrections to the first printing of

Olver, P.J., *Applications of Lie Groups to Differential Equations*, Second Edition, Springer–Verlag, New York, 1993.

Last updated: October 2, 2009

⋆⋆⋆ *On page 5, lines 24–27, change*

Thus T^2 can be covered by two coordinate charts

$$U_1 = \{(\theta,\rho) : 0 < \theta < 2\pi, 0 < \rho < 2\pi\},$$
$$U_2 = \{(\theta,\rho) : \pi < \theta < 3\pi, \pi < \rho < 3\pi\},$$

with overlap function . . .

to

Thus T^2 can be covered by three coordinate charts, e.g.

$$U_1 = \{(\theta,\rho) : 0 < \theta < 2\pi, 0 < \rho < 2\pi\},$$
$$U_2 = \{(\theta,\rho) : \pi < \theta < 3\pi, \pi < \rho < 3\pi\},$$
$$U_3 = \{(\theta,\rho) : \frac{1}{2}\pi < \theta < \frac{5}{2}\pi, \frac{1}{2}\pi < \rho < \frac{5}{2}\pi\}.$$

The first overlap function is . . .

⋆⋆⋆ *On page 10, line 6, change*

$\phi \circ \tilde{\phi}^{-1} : \mathbb{R} \to \mathbb{R}$

to

$\phi^{-1} \circ \tilde{\phi} : \mathbb{R} \to \mathbb{R}$

⋆⋆⋆ *On page 19, line 17, change*

$x \in V_0 = \{\, x : |\, x \,| < \frac{1}{2} \,\}$

to

$x \in V_0 = \{\, -1 < x < \frac{1}{3} \,\}$

⋆⋆⋆ *On page 36, line 9, change*

for all $\varepsilon, \theta \in \mathbb{R}$, $x \in M$, such that both sides are defined, if and only if

to

for all $x \in M$, and $(\varepsilon, \theta) \in V$, where $V \subset \mathbb{R}^2$ is a connected open subset containing $(0,0)$ such that both sides of (1.34) are defined at all points therein, if and only if

$\star\star\star$ *On page 37, lines 7–9, change*

...plane:

$$V = \{(\theta,\varepsilon) : \text{both sides of (1.34) are defined at}(\theta,\varepsilon)\}$$

and

$$U = \{(\theta,\varepsilon) : \text{both sides of (1.34) are defined and equal at}(\theta,\varepsilon)\}$$

to

...plane: first V is the connected component of

$$\widehat{V} = \{(\theta,\varepsilon) : \text{both sides of (1.34) are defined at}(\theta,\varepsilon)\}$$

containing the origin; second $U = \widehat{U} \cap V$, where

$$\widehat{U} = \{(\theta,\varepsilon) : \text{both sides of (1.34) are defined and equal at}(\theta,\varepsilon)\}$$

$\star\star\star$ *On page 37, line 10, delete the sentence*

Note that $U \subset V$, and that V is a connected subset of the (θ,ε) plane.

$\star\star\star$ *On page 37, line 14, add the following sentence after the final $U = V$.*

Warning: It is not, in general, true that $\widehat{U} = \widehat{V}$!

Remark: The preceding corrections are because Theorem 1.34 had a subtle flaw in it, first pointed out to me by James Devlin. The following exercise gives a counterexample to the original version. More details can be found in my paper: Olver, P.J., Non-associative local Lie groups, *J. Lie Theory* **6** (1996) 23–51. Thanks also to Hans Lundmark for comments.

Exercise: Let $M = \{\,(r,\theta) \mid r > 0\,\}$. Prove that the two vector fields

$$\mathbf{v} = \cos\theta\,\partial_r - \frac{\sin\theta}{r}\partial_\theta, \qquad \mathbf{w} = \sin\theta\,\partial_r + \frac{\cos\theta}{r}\partial_\theta,$$

commute, $[\mathbf{v},\mathbf{w}] = 0$ on M, but their flows do not globally commute. *Hint*: Consider r,θ as polar coordinates.

$\star\star\star$ *On page 64, line 10, delete the middle terms between the two = signs. Thus, the equation should read*

$$exp(\varepsilon\mathbf{v}_0)^*[\omega|_{exp(\varepsilon\mathbf{v}_0)x}] = \sum_I \alpha_I(e^{\varepsilon}x)e^{k\varepsilon}dx^I,$$

⋆⋆⋆ *On page 64, line 18, change*

$$\int_{\log\varepsilon}^{1}$$

to

$$\int_{\exp\varepsilon}^{1}$$

⋆⋆⋆ *On page 64, line 19, change*

$$\lambda = \log\widetilde{\varepsilon}$$

to

$$\lambda = e^{\tilde{\varepsilon}}$$

⋆⋆⋆ *On page 67, line 15, change*

sort

to

short

⋆⋆⋆ *On page 83, line 14, change*

folowing

to

following

⋆⋆⋆ *On page 93, line 8, change*

functon

to

function

⋆⋆⋆ *On page 113, line 4, change*

$$\xi = u$$

to

$$\xi = -u$$

⋆⋆⋆ *On page 123, line 5, change*

$$\tau\,\frac{\partial}{\partial\tau}$$

to

$$\tau\,\frac{\partial}{\partial t}$$

⋆⋆⋆ *On page 197, line -10, change*

SO(3)–invariant solutions exist.

to

SO(3)–invariant solutions can be constructed by this technique.

⋆⋆⋆ *On page 207, line -2, change*

Example 2.64

to

Example 2.44

⋆⋆⋆ *On page 280, in the table, change*

$I_x = x\,D - y\,A + \frac{1}{2}\,x\,u\,u_t + t\,M_x$

to

$I_x = x\,D + y\,A + \frac{1}{2}\,x\,u\,u_t + t\,M_x$

⋆⋆⋆ *Thanks to Gehrt Hartjen for checking through this table and the table on page 340 in his Mathematics Diplomarbeit in Aachen, 2001.*

⋆⋆⋆ *On page 285, line 5, change*

4.13. (a)

to

** 4.13. (a)

⋆⋆⋆ *On page 285, line 7, change*

Prove that the reduced system Δ/G for the G-invariant solutions of Δ is also the Euler-Lagrange equations for some variational problem on the quotient manifold M/G. Does this generalize to nonvariational symmetry groups?

to

Is the reduced system Δ/G for the G-invariant solutions of Δ necessarily the Euler-Lagrange equations for some variational problem on the quotient manifold M/G? See I.M. Anderson and M. Fels, Symmetry reduction of variational bicomplexes and the principle of symmetric criticality, *Amer. J. Math.* **119** (1997) 609–670, for details.

⋆⋆⋆ *On page 290, line -4, change*

Exercise 2.33

to

Exercise 2.35

⋆⋆⋆ *On page 323, line -8 change*

a third order evolution equation is integrable

to

a third order evolution equation in which u_{xxx} occurs linearly is integrable

⋆⋆⋆ *On page 328, line 10, change*

Bluman and Kumei, [3],

to

Bluman and Kumei, [2],

$\star\star\star$ *On page 340, in line 4 of the table, change*

$$-y\,u_{xxx} + x\,u_{xyy} + u_{xy}$$

to

$$-y\,u_{xxx} + x\,u_{xxy} + u_{xy}$$

$\star\star\star$ *On page 340, in line 5 of the table, change*

$$u_{xx}\left(y\,u_{yt} + \tfrac{1}{2}\,u_t\right) - u_{yy}\left(x\,u_{xt} + \tfrac{1}{2}\,u_t\right)$$

to

$$-u_{xx}\left(y\,u_{yt} + \tfrac{1}{2}\,u_t\right) + u_{yy}\left(x\,u_{xt} + \tfrac{1}{2}\,u_t\right)$$

$\star\star\star$ *On page 350, lines -8 to -7, change*

the their Fréchet

to

their Fréchet

$\star\star\star$ *On page 366, line -7, change*

$J \setminus I$

to

$I \setminus J$

$\star\star\star$ *Thanks to Rob Thompson for catching this and several other errors.*

$\star\star\star$ *On page 381, Exercise 3.16a:*

The system does not, in fact have a recursion operator, although there is a recursive formula for generating the higher order symmetries. On the other hand, the related system

$$u_t = u_{xx} + v^2, \qquad v_t = v_{xx},$$

does admit a recursion operator. Details can be found in: Beukers, F., Sanders, J.A., and Wang, J.P., On integrability of systems of evolution equations, *J. Diff. Eq.* **172** (2001), 396-408.

$\star\star\star$ *On page 381, Exercise 3.16b:*

A proof that the Bakirov system has only one generalized symmetry can now be found in: Beukers, F., Sanders, J.A., and Wang, J.P., One symmetry does not imply integrability, *J. Diff. Eq.* **146** (1998), 251–260.

$\star\star\star$ *On page 480, line 2 change*

Leipz. Berich. **1** (1895)

to

Leipz. Berichte **47** (1895)

$\star\star\star$ *On page 480, line 5 change*

Leipz. Berich. **3** (1897)

to

Leipz. Berichte **49** (1897)