Math 5588
January 27 2005

Homework Assignment # 1

1. A homogeneous, isotropic circular metal disk of radius 1 meter has its entire boundary insulated. The initial temperature at a point is equal to the distance of the point from the center. Formulate the initial-boundary value problem governing its temperature. What is the equilibrium temperature of the disk?

2. Solve the following initial-boundary value problem for the heat equation $u_t = 2\,\Delta u$ on the rectangle $-1 \le x \le 1, 0 \le y \le 1$, in which the two long sides are kept at $0°$, the two short sides are insulated, and the initial temperature distribution is $u(0,x,y) = \begin{cases} -1, & x < 0, \\ +1, & x > 0. \end{cases}$

3. Two square plates are made out of the same homogeneous material, and both are initially heated to $100°$. All four sides of the first plate are held at $0°$, while for the second plate one of its sides is insulated and the other 3 held at $0°$. Which plate cools down the fastest? How much faster? Assuming the thermal diffusivity $\gamma = 1$, how long do you have to wait until every point on the plate is within $1°$ of its equilibrium temperature? To answer the last question, for simplicity ignore the higher order terms in the Fourier expansion.

4. (a) Prove that the total heat $H(t) = \iint_\Omega u(t,x,y)\,dx\,dy$ is conserved (i.e., constant) for the homogeneous Neumann boundary value problem for the heat equation. (b) Use part (a) to prove that the equilibrium solution for the homogeneous Neumann boundary value problem is a constant, equal to the mean initial temperature $u(0,x,y)$. (c) What can you say about the total heat for the Dirichlet boundary value problem?

5. Let $A = A^T$ be a symmetric matrix. Prove that any two eigenvectors corresponding to distinct eigenvalues are orthogonal under the dot product.

Due: Tuesday, February 8

Text: Walter A. Strauss, *Partial Differential Equations: an Introduction*, John Wiley & Sons, New York, 1992.