Non-divergence elliptic equations of second order with singular drift

Mikhail Safonov
University of Minnesota
December 10, 2008

Consider the uniformly elliptic equation

$$Lu = \sum a_{ij} D_{ij} u + \sum b_i D_i u = 0 \qquad \text{in a domain} \quad \Omega \subset \mathbb{R}^n,$$

where a_{ij} and b_i are measurable functions. We discuss various properties of solutions u, such as the *interior and boundary Harnack inequalities* in the case $b_i \in L^n$, and the boundary upper and lower (Hopf type) estimates for the gradient Du in the case $b_i \in L^q$ with $q > n$. The assumptions on b_i and on the boundary are sharp even if $\sum a_{ij} D_{ij} u = \Delta u$. We demonstrate our techniques in a simplest case when $L = \Delta$ - the Laplacian, so that the whole exposition is going to be elementary.