Math 2263, Fall 2011
Quiz 11

Name: Solutions

Read each question carefully. Only scientific calculators can be used. You may not use your phone, ipod, graphing calculator, computer, etc. You have 15 minutes to take this 20 point quiz.

1. (*10 points*) Given the vector field $\mathbf{F} = xyz\mathbf{i} - y^2z\mathbf{j} + yz^2\mathbf{k}$, is there a vector field $\mathbf{G}$ such that $\nabla \times \mathbf{G} = \mathbf{F}$? [*hint:* gcd]

If there were a G such that $F = \nabla \times G$, then we would have $\operatorname{div}(F) = \operatorname{div}(\nabla \times G) = 0$, because applying curl followed by div always results in 0.

On the other hand,

$\operatorname{div}(F) = yz - 2yz + 2yz = yz$.

Since $yz \neq 0$, we conclude that

<u>no such G exists.</u>

2. (*10 points*) Use the Divergence Theorem to calculate the surface integral

$$\iint_S \mathbf{F} \cdot d\mathbf{S},$$

where S is the surface of the box enclosed by the planes $x = 0$, $x = a$, $y = \overset{0}{\cancel{a}}$, $y = b$, $z = 0$ and $z = c$, where a, b and c are positive numbers, and $\mathbf{F}(x, y, z) = x^2yz\mathbf{i} + xy^2z\mathbf{j} + xyz^2\mathbf{k}$.

By the divergence theorem,

$$\iint_S F \cdot dS = \iint_E \operatorname{div}(F)\, dV,$$

where E is the volume enclosed by the surface S.

Thus

$$\iint_S F \cdot dS = \int_0^a \int_0^b \int_0^c 2xyz + 2xyz + 2xyz \, dz\, dy\, dx$$

$$= 6\left(\int_0^a x\, dx\right)\left(\int_0^b y\, dy\right)\left(\int_0^c z\, dz\right) \quad \left\{ \begin{array}{l} \text{by Fubini's} \\ \text{theorem} \end{array} \right.$$

$$= 6\left(\tfrac{1}{2}a^2\right)\left(\tfrac{1}{2}b^2\right)\left(\tfrac{1}{2}c^2\right)$$

$$= \frac{3}{4}a^2b^2c^2$$