Math 2263, Fall 2011
Quiz 9

Name: Solutions

Read each question carefully. Only scientific calculators can be used. You may not use your phone, ipod, graphing calculator, computer, etc. You have 15 minutes to take this 20 point quiz.

1. (*7 points*) Let $\mathbf{F}(x,y,z) = P(x,y,z)\mathbf{i} + Q(x,y,z)\mathbf{j} + R(x,y,z)\mathbf{k}$ be a vector field on R^3. Show that $\nabla \cdot (\nabla \times \mathbf{F}) = 0$.

$$\nabla \times F = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ \partial_x & \partial_y & \partial_z \\ P & Q & R \end{vmatrix} = (R_y - Q_z)\mathbf{i} - (R_x - P_z)\mathbf{j} + (Q_x - P_y)\mathbf{k}$$

So

$$\nabla \cdot (\nabla \times F) = \frac{\partial}{\partial x}(R_y - Q_z) + \frac{\partial}{\partial y}(R_x - P_z) + \frac{\partial}{\partial z}(Q_x - P_y)$$

using Clairaut's theorem $\longrightarrow$ $= R_{xy} - Q_{xz} + R_{xy} - P_{yz} + Q_{xz} - P_{yz}$

$= 0$

2. (*6 points*) Is there a vector field $\mathbf{G}$ on R^3 such that

$$\mathbf{G} = \left\langle xyz, -y^2z, yz^2 \right\rangle?$$

Explain your answer.

I messed up the statement of this problem. As is, an appropriate answer would be:

Yes, let $G = \langle xyz, -y^2z, yz^2 \rangle$

(See Quiz 11 for the correct version)

3. (*7 points*) Evaluate

$$\int_C y^3 dx - x^3 dy$$

where C is the circle centered at the origin of radius 2. [Hint: use Green's theorem].

We have, by Green's theorem,

$$\int_C y^3 dx - x^3 dy = \iint_D -3x^2 - 3y^2 \, dA$$

where D is the disc of radius 2.

Converting to polar,

$$\iint_D -3(x^2+y^2)\, dA$$

$$= \int_0^{2\pi}\int_0^2 -3r^2 \, r\,dr\,d\theta$$

$$= -3\int_0^{2\pi}\int_0^2 r^3\,dr\,d\theta$$

$$= -3\int_0^{2\pi} \frac{1}{4}r^4\Big|_0^2 \, d\theta$$

$$= -\frac{3}{4}\int_0^{2\pi} 16\, d\theta$$

$$= -\frac{3}{4}(16)(2\pi)$$

$$= -24\pi$$