Lie groups and Lie algebras
Math 8271–8272 (Fall 2002 – Spring 2003)

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Course meets: MWF, 11:15 a.m.–12:05 p.m., Vincent 6

Basic prerequisites for this course are the knowledge of linear algebra, a course in abstract algebra, real analysis (including elements of point-set topology) and familiarity with basic ideas of the theory of ordinary differential equations. We will avoid using sophisticated apparatus of differential geometry, developing whatever becomes necessary as we move along.

Course objectives.

1. Introduce Lie groups and Lie algebras, learn to operate with them in concrete terms (matrix groups, matrices, differential operators).
2. Recognize Lie groups and Lie algebras within basic mathematical subjects, such as:
   - *linear algebra* (e.g. reduction theory for matrices deals with various decompositions in the group GL_n);
   - *differential equations* (e.g. the phase flow of a first order ODE is a one-parametric group action);
   - *geometry* (e.g. the sphere S^2 is a homogenuous space for the orthogonal group $SO(3)$).
3. Exploit connections between Lie theory and "basic subjects" in both directions:
   - use basic subjects to establish properties of Lie groups and Lie algebras (e.g. matrix exponent gives rise to the exponential map, classification of bilinear forms up to conjugacy leads to various classical groups, etc), *and*
   - exploit the structural framework of Lie Theory to conceptualize constructions in basic subjects (e.g. Erlanger Program of Klein, first order linear systems of partial differential equations, etc).
4. Develop foundations of Lie Theory. Establish connections between Lie groups and Lie algebras.
5. Develop structure theory of Lie groups and Lie algebras and derive various classification results (simple Lie algebras, compact Lie groups, reflection groups in a Euclidean space, Levi–Mal'cev theorem).
6. Sample applications of Lie Theory, possibly working out some of them in detail (for instance, *representation theory and its connections with combinatorics*).

The textbook for this course is

Wulf Rossmann, *Lie Groups: An Introduction Through Linear Groups,* Oxford Graduate Texts in Mathematics, No.5, Oxford University Press, 2002 (ISBN 0-19-859683-9).

As a reference for results on Lie algebras we will use the well-known text by

James Humphreys, *Introduction to Lie Algebras and Representation Theory,* Graduate Texts in Mathematics, No.9, Springer (ISBN 3-540-90053-5 hardcover).

For complementary reading I would recommend two surveys on Lie theory:

Roger Howe, *Very Basic Lie Theory,* American Mathematical Monthly, 90, November 1983, pp. 600–623,
Roger Howe, *A Century of Lie Theory,* in American Mathematical Society Centennial Publications, Volume II, pp. 101–320.

The first paper is a condensed account of foundations of Lie Theory, in the same spirit as Rossmann's textbook, whereas the second is a comprehensive account of ideas and developments of Lie Theory that starts in an elementary fashion but brings the reader to the cutting edge of the modern research.

Course policies.

- There will be a moderate length homework assignment due every week.
- I expect that you will think over the course material at home and discuss it in class.
- Everyone will be assigned an individual project and will make a presentation at the end of the semester. The attendance, completeness of the homework, and project/presentation will determine the grade.