# DIAGNOSTIC EXAM: ARE YOU READY FOR MATH 5651?

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In Math 5651 we use every scrap of high school algebra and freshman calculus. We also use a substantial portion of sophomore calculus, mainly the part having to do with double integrals over regions with possibly complicated shapes, and the formula for evaluating double integrals by substitution.

Answer all of the following questions. If you have trouble doing so, you are sure to have a tough time in the course, unless you budget extra time for buckling down to review basic skills.

## Problem 0

What does $n!$ mean?

## Problem 1

Five people are seated in a row. Two of them are smiling and the remaining three are frowning. How many different ways can the smiles and frowns be arranged?

## Problem 2

Five slips of paper numbered 1 through 5 are placed in a hat. Person A draws a slip of paper from the hat. Then person B draws a slip of paper. Finally person C draws a slip of paper. How many different ways can persons A,B,C draw the numbered slips of paper?

## Problem 3

Explain why

$$\sum_{i=1}^{10} i^2 = \sum_{x=5}^{15} (x^2 - 10x + 25)$$

without evaluating the sums on either side.

## Problem 4

Explain why

$$\int_0^\infty e^{-x} dx = \int_0^1 (-\log x) dx$$

without evaluating the integrals on either side.

## Problem 5

Evaluate $\int_{-\infty}^{\infty} e^{-2x^2+7x} dx$.

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## Problem 6

Evaluate $\int_0^{1/3} x(1-x)^2 dx$.

## Problem 7

Evaluate $\int_0^1 (x-y)^2 y^2 dy$. (The answer will be a function of $x$.)

## Problem 8

Evaluate $\int_0^\infty xe^{-7x} dx$.

## Problem 9

Let

$$R = \{(x, y) \in \mathbb{R}^2 \mid x, y \geq 0,\ x + 2y \leq 1\}.$$

Evaluate the double integral

$$\int\int_R xydxdy$$

in two ways. In the first way, integrate over $x$ first. In the second way, integrate over $y$ first.

## Problem 10

Find the centroid of the region $\{(x, y) \in \mathbb{R}^2 \mid 0 \leq y \leq x^2,\ 0 \leq x \leq 1\}$.

## Problem 11

Find the terms through sixth order of the Taylor expansion at $x = 0$ of the function $\sqrt{1 + x^2}$.

## Problem 12

Find the infinite sum

$$\sum_{n=1}^\infty n \left(\frac{2}{3}\right)^n.$$

## Problem 13

Consider the regions

$$R = \{(x, y) \in \mathbb{R}^2 \mid 1 \leq x \leq 2,\ 3 \leq y \leq 4\}$$

and

$$S = \{(u, v) \in \mathbb{R}^2 \mid 1/v \leq u \leq 2/v,\ 3 \leq v \leq 4\}.$$

Sketch the regions $R$ and $S$ on different axes. Explain why the transformation

$$\begin{aligned} u &= x/y \\ v &= y \end{aligned}$$

carries the region $R$ to the region $S$. Use the Jacobian derivative formula for transforming double integrals to explain why

$$\int\int_R xy^2 dxdy = \int\int_S uv^4 dudv$$

without evaluating the integrals on either side. Check your work by evaluating the integrals on both sides directly.