Math 5711 Combinatorial optimization
Spring 2004, Vic Reiner
Midterm exam 1- Due Wednesday February 25, in class

Instructions: This is an open book, open library, open notes, open web, take-home exam, but you are *not* allowed to collaborate. The instructor is the only human source you are allowed to consult.

1. (15 points) Schrijver Problem 1.3 on p. 14. Draw the associated directed graph, and the last Bellman-Ford function on its vertices.

2. (15 points) In Schrijver's knapsack problem example (Application 1.3), assume that object 2 is no longer available, so one has

article	volume	value
1	5	4
3	2	3
4	2	5
5	1	4

Solve this new knapsack problem by drawing an appropriate directed graph and using the Bellman-Ford algorithm.

3. (15 points) Give an example of a directed graph $D = (V, A)$ and a length function on the arcs $\ell : A \to \mathbb{Z}$ with these properties:

- $|V| = 3$, i.e. there are exactly 3 vertices, labelled $V = \{s, r, t\}$,
- there are no directed cycles of negative length,
- Dijkstra's algorithm fails to find an $s - t$ directed path of minimum length, but the Bellman-Ford algorithm works. (Write down the output from both algorithms)

4. Consider the following LP problem:

$$\text{minimize } x_2$$

$$\text{subject to} \quad \begin{aligned} x_1 + x_2 &\geq 1 \\ 3x_1 + 2x_2 &\leq 6 \\ x_1, x_2 &\geq 0 \end{aligned}$$

(a) (5 points) Rewrite this problem in Chvátal's standard form.

(b) (20 points) Solve this problem using the two-phase simplex method. Be sure to write down each dictionary in both Phase I and II, and the entering/leaving variables at each pivot step. You do not need to show the algebra used in rewriting the dictionaries.

5. (15 points) Chvátal problem 1.5 on page 10.

6. (15 points) Chvátal problem 3.10 on page 44.