MATH 3592H EXAM REVIEW I

Monday, December 14, 2015

Name: ________________________________

Instructions: You may not use notes, textbooks, or personal electronic devices. As a courtesy to other students, please turn off all cell phones. Remember to include precise justification for your reasoning and show your work. Answers without any work or explanation will receive little or no credit. There are 3 questions and a 50 minute time limit on this exam. Good luck.

Question	Score	Maximum
1		31
2		33
3		35
Total		99

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1. Is the function

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \begin{cases} \frac{e^{x^2y^2}-1}{x^2+y^2} & (x,y) \neq (0,0) \\ 0 & (x,y) = (0,0) \end{cases}$$

differentiable at $(x,y) = (0,0)$? Justify your answer.

Follow up: What if we replace the numerator by $e^{xy^2} - 1$ or $(e^{x^2} - 1)\sin y$?

Alternate question: Use the definition of the derivative to show that the following function is differentiable everywhere:

$$f(x,y) = xy + y^2 - 5$$

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2. a) Recall that the row space $R(A)$ of a matrix A is the span of its row vectors. Show that $R(A) = N(A)^{\perp}$. Here $V^{\perp}$ denotes the set of vectors orthogonal to ALL $v \in V$.

   b) Show that $N(A^{\top}) = C(A)^{\perp}$, where $C(A)$ is the column space of A.

   c) Find a basis for each of $N(A), C(A), R(A), N(A^{\top})$ if

$$A = \begin{pmatrix} 1 & 1 & 2 & 0 & 0 \\ 0 & 1 & 1 & -1 & -1 \\ 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 3 & -1 & -3 \end{pmatrix}$$

Follow up: Given an $m \times n$ matrix A of rank r, what are the dimensions of $R(A), N(A), C(A), N(A^{\top})$?

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3. Consider the set of solutions to the system of equations

$$x^3 + 2xy - z = 0$$
$$y^2 - xz + x = 0$$

a) Does this system of equations define a smooth 1-manifold in $\mathbb{R}^3$?

b) Find the equation of the tangent plane to the locus at $(x, y, z) = (1, 0, 1)$.

c) At $(1, 0, 1)$, what are the possible independent variables defining an implicit function ϕ locally parametrizing the zero locus? Pick one and give the second order Taylor expansion for the resulting function ϕ at $(1, 0, 1)$.

Alternate question: Similar for $F : \mathbb{R}^5 \longrightarrow \mathbb{R}^3$ given by

$$F(x_1, x_2, x_3, x_4, x_5) = (2x_1 + x_2 + x_3 + x_4 - 1, x_1x_2^3 + x_1x_3 + x_2^2x_4^2 - x_4x_5, x_2x_3x_5 + x_1x_3^2 + x_4x_5^2)$$