Name: ______________________________

Signature: ______________________________

Section and TA: ______________________________

Math 1272. Lecture 010 (V. Reiner) Midterm Exam I
Thursday, February 18, 2010

This is a 50 minute exam. No books, notes, calculators, cell phones or other electronic devices are allowed. There are a total of 100 points. To get full credit for a problem you must show the details of your work. Answers unsupported by an argument will get little credit. Do all of your calculations on this test paper.

Problem	Score
1.	
2.	
3.	
4.	
Total:	

Problem 1. *(30 points total; 15 points each) Compute the following indefinite integrals.*

a.

$$\int \frac{x^2 dx}{x^2 - 3x + 2}$$

b.

$$\int \frac{\arctan(\ln(x))dx}{x}$$

Problem 2. *(30 points total; 15 points each) Compute the following definite or improper integrals.*

a.

$$\int_0^2 \frac{x^5 dx}{\sqrt{4 - x^2}}$$

b.

$$\int_1^{+\infty} \frac{dx}{(3x+2)^2}$$

Problem 3. *(25 points total) Write down as a sum the estimate for* $\int_1^5 \sin(e^x)dx$ *using* 4 *equal subintervals, via the following numerical integration rules, but do not attempt to simplify or numerically evaluate them:*

a. (8 points) *midpoint rule,*

b. (8 points) *trapezoidal rule,*

c. (9 points) *Simpson's rule,*

Problem 4. *(15 points) Consider the surface of revolution obtained by rotating around the x-axis the portion of the curve $y = 2\sqrt{1+x}$ that lies between $x = 1$ and $x = 2$. Note that this is a* **surface** *of revolution,* **not** *a volume of revolution.*

a. (6 points) *Write down a definite integral which calculates its surface area, but do not evaluate this integral yet.*

b. (9 points) *Evaluate the integral from your answer to part (a).*

Brief solutions

Problem 1(a) (15 points)

$$\int \frac{x^2 dx}{x^2 - 3x + 2} = \int \left(1 + \frac{3x-2}{x^2-3x+2}\right) dx$$

by long division

$$= x + \int \frac{3x-2}{(x-1)(x-2)} dx$$

$$= x + \int \left(\frac{-1}{(x-1)} + \frac{4}{x-2}\right) dx$$

by partial fractions

$$= x - \ln|x-1| + 4\ln|x-2| + C$$

Problem 1(b) (15 points)

$$\int \frac{\arctan(\ln(x))dx}{x} = \int \arctan(y)dy$$

by substituting $y = \ln(x), dy = \dfrac{dx}{x}$

$$= y\arctan(y) - \int \frac{y}{1+y^2} dy$$

by integration by parts with $u = \arctan(y), dv = dy$

$$= y\arctan(y) - \frac{1}{2}\ln|1+y^2| + C$$

$$= \ln(x)\arctan(\ln(x)) - \frac{1}{2}\ln|1+\ln(x)^2| + C$$

Problem 2(a) (15 points)

$$\begin{aligned}
\int_0^2 \frac{x^5dx}{\sqrt{4-x^2}} &= \frac{1}{2}\int_0^2 \frac{x^5dx}{\sqrt{1-\frac{x^2}{4}}} \\
&= \frac{1}{2}\int_{\theta=0}^{\theta=\frac{\pi}{2}} \frac{(2\sin(\theta))^5}{\sqrt{1-\sin^2(\theta)}} 2\cos(\theta)d\theta \\
&\text{by trig substitution } \frac{x}{2}=\sin(\theta), dx = 2\cos(\theta)d\theta \\
&= 32\int_0^{\frac{\pi}{2}} \sin(\theta)^5 d\theta \\
&= 32\int_0^{\frac{\pi}{2}} \left(\sin(\theta)^2\right)^2 \sin(\theta)d\theta \\
&= 32\int_0^{\frac{\pi}{2}} \left(1-\cos(\theta)^2\right)^2 \sin(\theta)d\theta \\
&= 32\int_{u=1}^{u=0} \left(1-u^2\right)^2(-du) \\
&= 32\int_{u=0}^{u=1} (1-2u^2+u^4)du \\
&= 32\left[u-\frac{2}{3}u^3+\frac{1}{5}u^5\right]_0^1 \\
&= 32(1-\frac{2}{3}+\frac{1}{5})
\end{aligned}$$

Problem 2(b) (15 points)

$$\begin{aligned}
\int_1^{+\infty} \frac{dx}{(3x+2)^2} &= \lim_{b\to+\infty}\int_1^b \frac{dx}{(3x+2)^2} \\
&= \lim_{b\to+\infty}\int_{u=5}^{u=3b+2} \frac{\frac{1}{3}du}{u^2} \\
&\text{by substitution of } u=3x+2, du=3dx \\
&= \frac{1}{3}\lim_{b\to+\infty}\left[\frac{-1}{u}\right]_5^{3b+2} \\
&= \frac{1}{3}\lim_{b\to+\infty}\left(\frac{-1}{3b+2}-\frac{-1}{5}\right) \\
&= \frac{1}{3}(0+\frac{1}{5}) \\
&= \frac{1}{15}.
\end{aligned}$$

Problem 3 (25 points total) Write down as a sum the estimate for $\int_1^5 \sin(e^x)dx$ using 4 equal subintervals, via the following numerical integration rules, but do not attempt to simplify or numerically evaluate them:

a. (8 points) midpoint rule

$$1 \cdot \left(\sin(e^{3/2}) + \sin(e^{5/2}) + \sin(e^{7/2}) + \sin(e^{9/2})\right)$$

b. (8 points) trapezoidal rule

$$\frac{1}{2} \cdot \left(\sin(e^1) + 2\sin(e^2) + 2\sin(e^3) + 2\sin(e^4) + \sin(e^5)\right)$$

c. (9 points) Simpson's rule

$$\frac{1}{3} \cdot \left(\sin(e^1) + 4\sin(e^2) + 2\sin(e^3) + 4\sin(e^4) + \sin(e^5)\right)$$

Problem 4 (15 points) Consider the surface of revolution obtained by rotating around the x-axis the portion of the curve $y = 2\sqrt{1+x}$ that lies between $x = 1$ and $x = 2$. Note that this is a **surface** of revolution, **not** a volume of revolution.

a. (6 points) Write down a definite integral which calculates its surface area, but do not evaluate this integral yet.

$$\begin{aligned} 2\pi \int_1^2 f(x)\sqrt{1+f'(x)^2}dx &= 2\pi \int_1^2 2\sqrt{1+x}\sqrt{1+\left(2\cdot\frac{1}{2}(1+x)^{-\frac{1}{2}}\right)^2}dx \\ &= 4\pi \int_1^2 \sqrt{1+x}\sqrt{1+\frac{1}{1+x}}dx \\ &= 4\pi \int_1^2 \sqrt{1+x+1}dx \\ &= 4\pi \int_1^2 \sqrt{x+2}dx \end{aligned}$$

b. (9 points) Evaluate the integral from your answer to part (a).

$$4\pi \int_1^2 \sqrt{x+2}dx = 4\pi \int_3^4 \sqrt{u}du$$

by substitution of $u = x + 2, du = dx$

$$= 4\pi \left[\frac{2}{3}u^{\frac{3}{2}}\right]_3^4$$

$$= \frac{8\pi}{3}(4^{\frac{3}{2}} - 3^{\frac{3}{2}})$$

$$= \frac{8\pi}{3}(8 - 3^{\frac{3}{2}})$$