Referee’s report on
“Noncrossing Partitions for the group \(D_n\)”
by C. Athanasiadis and V. Reiner

In this paper the authors derive some properties of a type \(D\) analogue of the noncrossing partition lattice. This analogue has the advantage that it fits nicely into a general framework about noncrossing and nonnesting partitions for general Coxeter groups. The paper is also very well written, so it should definitely be published. I only have some minor suggestions to help the exposition and correct some small problems. In the list that follows p/l (respectively, p//l) refers to the lth line from the top (respectively, bottom) of page p.

3//13–15 Here or elsewhere I think it would be worth mentioning that Nathan Reading has also come up with yet a third type \(D\) analogue of \(NC^A(n)\).

5/4 You are using length here as length in a symmetric group rather than length in \(D_n\), which you are about to define. To avoid confusion, I suggest you say “number of elements” at this juncture.

5/18 A single balanced cycle is not an element of \(D_n\), and so does not have a length as a product of \(D_n\)-reflections. You could say that such a cycle contributes \(k\) to the length of the full permutation.

7/16 Since nonnesting partitions are a relatively new concept vis-a-vis noncrossing ones, I think it would be worth giving the motivation for the name here in addition to referring the reader to Postnikov. This is easy to do by taking a 2-element chain in the root poset for \(A_n\) and noticing how the corresponding partition has a pair of nesting blocks.

8/13 I don’t like this definition of crossing, especially since I think of a point as having empty relative interior and so it would never intersect another block. I suggest instead that you say that \(B\) and \(B’\) cross if the relative interior of one of the blocks contains a point of the other.

10/5–6 There is no reason to say that one block contains \(n\) and then other \(-n\) since that is automatic because the zero block always contains this pair. If you want to emphasize that in passing from \(y\) to \(x\) one completely removes the zero block, I suggest that you start the phrase “splitting the whole zero block of \(y\).”

10//16 I see no reason to introduce these infinite cyclic expressions when you already have circles labeled with a finite number of elements so as to achieve the same effect.

12//14 So as to emphasize that this labeling relates directly to what was talked about in the previous sentence (rather than just being the next thing one does in a sequence of events) I would replace “Then” by “In this case.”

12//13 I would remove the comma as not necessary and possibly confusing since the reader might assume it was part of the labeling.

12//2 You want “multichain” rather than “chain.”

13/15 I would write \(1 + s_i\) in the more standard order \(s_i + 1\) as you have been doing for other expressions like \(#L + 1\).

16/3–5 I think it would be conceptually cleaner to amalgamate the last two cases into one by
saying “Then replace them with $B \cup (-B) \setminus \{n, -n\}$ which removes the two blocks if they are singletons or replaces them by a zero block if they are not.”

16/8 and 10 Here I would use $\sqcup$ to emphasize that you are taking the disjoint union of the two partitions (where multiplicities add) rather than the union (where one takes the maximum multiplicity).

18/1–2 For better flow, I would put this comment after Corollary 5.5 and its proof. In addition, “follows also” should be “also follows.”

18/6 You don’t need to cite 5.2 in the proof as 5.4 and 1.2 (i) suffice.

18/1 Here and in several other places you should use $\subseteq$ (as you do, e.g., on 11/5) since you can have equality.

22/8 You need to work in the formal power series ring in these variables, not just the polynomial ring, otherwise expressions like you have on 23/27 are not well defined.