Box-ball systems

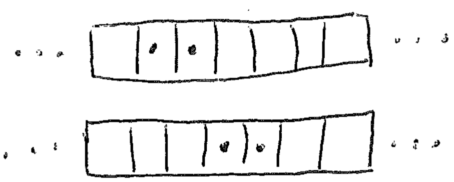
infinite number of boxes
finite number of balls

carrier

1) If the carrier passes a ball, they pick it up.
2) If the carrier passes an empty box while holding a ball, they put a ball in the box.

Ex: t = 0

t = 1



Def: A series of balls with no empty boxes in between is a <u>soliton</u>. Its <u>amplitude</u> is the number of balls in it.

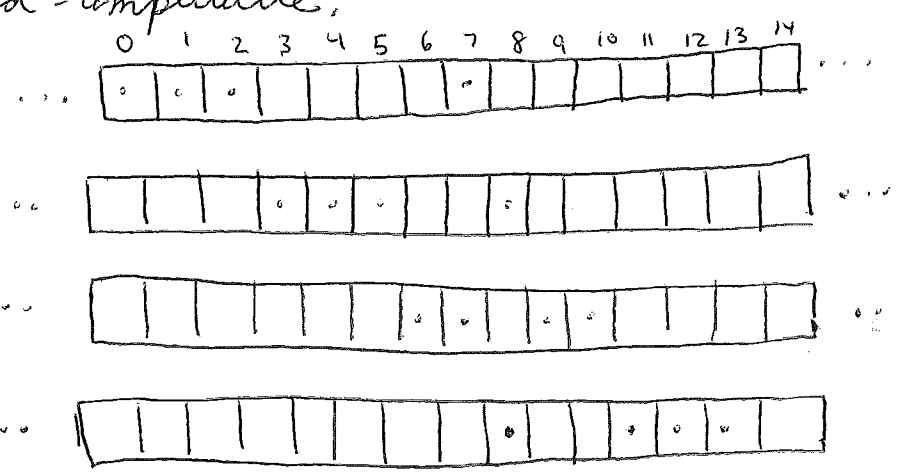
<u>Remark</u>: If there's no interference, a soliton moves with speed = amplitude.

Ex: t = 0

t = 1

t = 2

t = 3



3	1
0	7
3	8
11	8

Exercise: When a soliton of amplitude n passes a soliton of amplitude m, they resolve into solitons of amplitudes m, n.

Exercise: After a collision of solitons, the larger soliton is pushed to the right and the smaller soliton is pushed to the left.

We would like to have a notion of speed for the whole system such that the speed of a system is constant

Idea: avg pos. of balls

Ex: $\frac{3+4+5+8}{4} - \frac{0+1+2+7}{4} = \frac{5}{2}$

$\frac{6+7+9+10}{4} - \frac{3+4+5+8}{4} = 3$ X

Idea 2: avg pos. of solitons

Ex: $\frac{4+8}{2} - \frac{1+7}{2} = 2$

$\frac{6.5+9.5}{2} - \frac{4+8}{2} = 2$

$\frac{8+12}{2} - \frac{6.5+9.5}{2} = 2$

Exercise: Prove this idea of speed gives a constant for any box-ball system.

<u>Generalized Box-Ball System</u>

We can generalize the box-ball system to have multiple kinds of balls.

For notation, we'll consider "empty" to be a "ball of type 1".

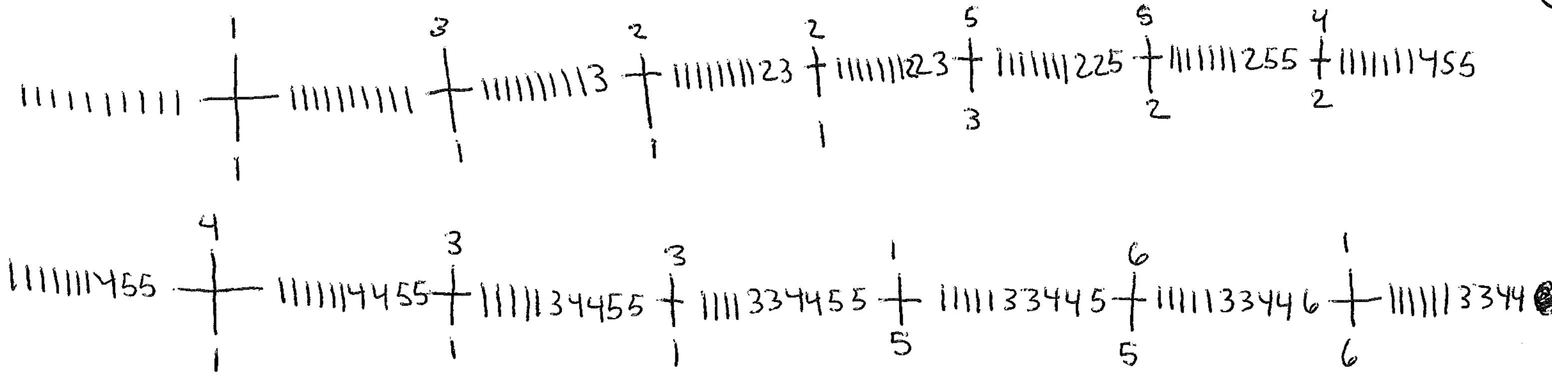
Let the carrier have a bag with space for K balls, where K = # balls of type >1 in the system.

1) The carrier starts with all balls of type 1 in the bag.

2) As the carrier passes a ball of type X, it replaces it with the largest ball of type < X in the bag. If there are no balls of type < X, the carrier replaces it with the largest ball in the bag.

Ex: t=0 ...1322554433161111...

K=10



$t=1$ ... 1111322111556443311l...

Another way to think about generalized box-ball systems:

Assuming there are n types of balls, we transition from time t to $t+1$ as follows:

1) Move the leftmost n to the nearest right empty space.
2) Repeat with the next leftmost n, then the next, until all n's have been moved once.
3) Repeat ①, ② with $n-1$, then $n-2$, through 2.

Ex: $t=0$

...1322554433161111...

...13225544331161111...

...132211443355611111...

...1322111133556 4411...

...112231111155644 33...

...1111322111556 4433...

Exercise: Show that the two methods of transitioning from time t to $t+1$ are the same.

Def: a <u>soliton</u> is a sequence of balls in decreasing order with no 1's.

<u>Conjecture</u>: The avg pos. of solitons gives a constant speed for generalized box-ball systems?

<u>Problem</u>: Prove or disprove the conjecture.