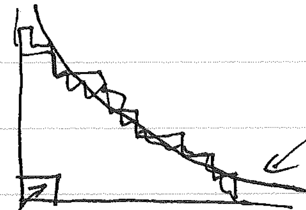
CPS 11/14/2014

R. Stanley

$\mathrm{Par}(n) :=$ partitions of n

$P : \mathrm{Par}(n) \longrightarrow [0,1]$ a probability distribution

e.g. uniform distribution $P(\lambda) = \frac{1}{p(n)}$

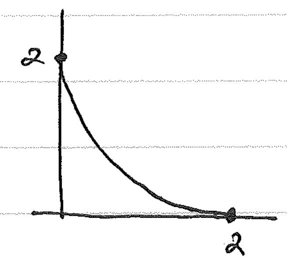


limiting shape

$e^{-cx} + e^{-cy} = 1$ (Vershik)

normalize so one square has area 1

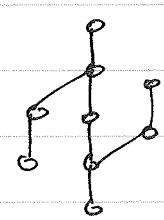
e.g. $P(\lambda) = \frac{(f^\lambda)^2}{n!}$ gives another shape for limiting curve:



Another one: given a poset

$\lambda_1 = 5$

$\lambda_1 + \lambda_2 = 8$



let $\lambda_1 := \max\{|C| : C \text{ a chain}\}$

$\lambda_1 + \lambda_2 := \max\{|C_1 \cup C_2| : C_i \text{ chains}\}$

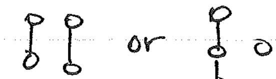
$\vdots$

THM (Greene-Kleitman) $\lambda_1 \geq \lambda_2 \geq \ldots$ is a partition

and $\lambda_1' + \lambda_2' + \ldots + \lambda_k' := \max\{|A_1 \cup \ldots \cup A_k| : A_i \text{ antichain}\}$

So if we pick a class $\mathcal{P}$ of posets and ask what is the limiting distribution on $\mathrm{Par}(n)$?

e.g. $\mathcal{P}$ = semiorders (unit interval orders) on n elements,

i.e. no induced  or

2+2 3+1

<u>Q</u>: Pick a semiorder P on $[n]$ with uniform distribution, and look at $\lambda(P)$. Is there a limiting distribution?

E. Engel observed that for semiorders one can construct the G-K partition $\lambda(P)$ greedily, so this might help.