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<u>Question:</u> We are Given ~~a fan $\mathcal{F}_n$~~ a central hyperplane arrangement $\subset \mathbb{R}^{n^2-n}$

with a lineality space of dim n

① Count # of chambers

② S_n acts on $\mathcal{F}_n$; count # chambers up to S_n-action

<u>The normal vectors</u>

$A \in \{0,1\}^{n\times n} \longleftrightarrow$ directed graph on n vertices, no loops

$A_{ij} = 1 \Longleftrightarrow i \longrightarrow j$

$A_{ii} = 0$

flatten $\rightsquigarrow$ vectors in $\{0,1\}^{n^2-n}$

e.g. $n=4$

$$\begin{array}{c|c} & \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \end{array} & \begin{bmatrix} \cancel{0} & 1 & 0 & 0 \\ 0 & \cancel{0} & 0 & 0 \\ 0 & 0 & \cancel{0} & 1 \\ 0 & 0 & 0 & \cancel{0} \end{bmatrix} \end{array} \longleftrightarrow \quad 1 \longrightarrow 2, \quad 3 \longrightarrow 4$$

Consider {bipartite graphs with m sources, m sinks, m edges}

$2 \le m \le \lfloor \frac{n}{2} \rfloor$

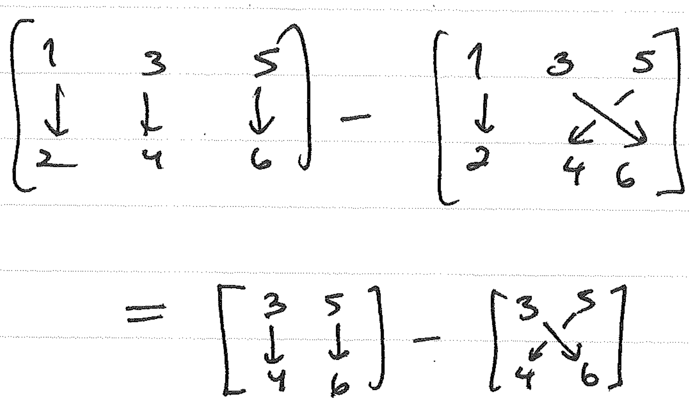
e.g. $n=4$

$\{$ $1\to 4,\ 2\to 3$, $1\to 3,\ 2\to 4$, $1\to 2,\ 3\to 4$, $1\to 4,\ 3\to 2$, $1\to 2,\ 4\to 3$, $1\to 3,\ 4\to 2$,

$2\to 1,\ 3\to 4$, $2\to 4,\ 3\to 1$, $2\to 1,\ 4\to 3$, $2\to 3,\ 4\to 1$, $3\to 1,\ 4\to 2$, $3\to 2,\ 4\to 1$ $\}$

$$\left\{\begin{array}{c}\text{normal vectors}\\ \text{for } \mathcal{F}_n\end{array}\right\} := \left\{ \underbrace{G_1 - G_2}_{} \in \mathbb{R}^{n^2-n} : (G_1, G_2) \text{ have same set of sources and sinks} \right\}$$

e.g.



e.g. for $n=4$, $\mathcal{F}_4$ has 6 normal vectors

Data:	$n=4$	$n=5$ (30 hyperplanes)	$n=6$
#chambers	62	3.5 million	?
#chambers up to S_n-action	6	~ 27000	?

For $n=4$, with 6 hyperplanes in $\mathbb{R}^{12}$
there is more lineality, and it ends up essential in $\mathbb{R}^5$