

1. State and prove the Schwarz inequality.
2. Find the projection of the vector $b = (0, 3, 0)$ onto the line of the vector $a = (2, 2, -1)$. In other words, find a scalar γ such that the vector $b - \gamma a$ is orthogonal to a . (Then the projection is γa .)
3. Let the matrix A be given by,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -6 & -2 \\ -2 & 7 & 5 \end{bmatrix}.$$

- (a) (20 pts.) Find an orthonormal basis for the column space of A
- (b) (20 pts.) Next, let the vector b be given by,

$$b = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}.$$

Find the orthogonal projection of this vector b onto $\text{col}(A)$, the column space of A

4. Let the matrix A of the previous problem be replaced by the matrix:

$$A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \end{bmatrix}.$$

Then find the projection of the vector b of the previous problem onto the column space of this matrix.

5. Let the function $y(x)$ be equal to 1 for $-\pi < x < 0$ and equal to -1 for $0 < x < \pi$. Find the first three Fourier coefficients of the function y . In other words, write

$$y(x) = a_0 + a_1 \cos x + b_1 \sin x + a_2 \cos 2x + \dots \quad (1)$$

Then find b_6 .

6. Let A be a given matrix and let λ be a given number. Define that λ is an eigenvalue of A .

7. Find the eigenvalues and eigenvectors of the matrix:

$$A = \begin{bmatrix} 3 & 4 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & 0 \end{bmatrix}.$$

8. Let A be a given matrix and let λ_1 and λ_2 be two eigenvalues of A such that $\lambda_1 \neq \lambda_2$. Prove that the corresponding eigenvectors are linearly independent.

9. Diagonalize the matrix A of Problem 7. That is to say, find an invertible matrix S and a diagonal matrix Λ such that

$$A = S\Lambda S^{-1}$$

10. Let A denote the matrix of Problem 7. Show that every solution of the system of differential equations,

$$\frac{dx}{dt}(t) = Ax(t),$$

is bounded in the variable t .

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