Quiz 4/ Week 6

3262, Spring '95, May 4 — Solutions

Problem 1

Again, this is not true. Take for example $a_n = 1/n, b_n = 1/n$. Then $\sum_1^\infty a_n$ and $\sum_1^\infty b_n$ both diverge as they are the harmonic series, but $\sum_1^\infty a_n b_n = \sum_1^\infty 1/n^2$ converges by the p-test.

Problem 2

For all positive y we have $\sin y < y$ so that in particular $\sin(1/x^2) < 1/x^2$ for all nonzero x. As $(47.11)^2 > 2/\pi$, we also have $1/x^2 < \pi/2$ for all x that occur in the integral and therefore $\sin 1/x^2 > 0$. It follows from the comparison test that $\int_{47.11}^\infty \sin(1/x^2)dx$ will converge if $\int_{47.11}^\infty 1/x^2 dx$ converges. But from the p-test we know that this is the case.

Problem 3

The function $(x \ln x)^{-1}$ is for $x \geq 2$ decreasing, positive and differentiable. So we can use the integral test to conclude that $\sum_2^\infty \frac{1}{n \ln n}$ is finite if and only if $\int_2^\infty \frac{1}{x \ln x} dx$ is finite.

Substitute $u = \ln x$ so that $du = \frac{dx}{x}$. Then $\int_{x=2}^{x=\infty} \frac{dx}{x \ln x} = \int_{x=2}^{x=\infty} \frac{du}{u} = \ln(u)|_{x=2}^{x=\infty} = \ln \ln x|_2^\infty = \infty$. Therefore $\int_2^\infty \frac{dx}{x \ln x}$ does not converge and hence $\sum_2^\infty \frac{1}{n \ln n}$ is not finite either.

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