1. Matching: k; a; f; h; e; c; j.

2 Short Answer: 0 (it is impossible); $\frac{1}{6}$; $\frac{1}{6}$ (The 100^{th} roll is independent of the first 99); $\frac{6000}{1000} = 0.60$.

3. True/False: F (the population decreases if $r < 1$); F (it should be about 5,000); T.

4. (a) ${}_7C_3 = \frac{7\times6\times5}{3\times2\times1} = 35$

(b) ${}_7P_3 = 7 \times 6 \times 5 = 210$

(c) Each letter is different, so this is really asking: how many different ways can we rearrange 4 letters? The answer is $4! = 4 \times 3 \times 2 \times 1 = 24$.

5. (a) $N = 36$

(b) $E = \{(4, 6), (5, 5), (6, 4)\}$

(c) $P(E) = \frac{\text{\# of outcomes in } E}{N} = \frac{3}{36} = \frac{1}{12}$

(d) We know $P(A \text{ wins}) + P(B \text{ wins}) + P(C \text{ wins}) + P(D \text{ wins}) = 1$. So

$$\begin{aligned} P(D \text{ wins}) &= 1 - P(A \text{ wins}) - P(B \text{ wins}) - P(C \text{ wins}) \\ &= 1 - 0.15 - 0.15 - 0.20 \\ &= 0.50 \end{aligned}$$

6. (a-i) $P_0 = 4$ because I own 4 power tools to begin with.

(a-ii) In general, $P_N = P_0 + N \times d$, so we have $P_N = 4 + N \times 5$.

(a-iii) $P_20 = 4 + 20 \times 5 = 4 + 100 = 104$.

(b) The formula for summing an arithmetic sequence is:

$$A_0 + A_1 + A_2 + \cdots + A_{N-1} = \frac{(A_0 + A_{N-1}) \times N}{2}.$$

In our case, $A_0 = 1$, $A_{N-1} = 1000$, and $N = 1000$ (because there are 1000 numbers in the list). So:

$$1+2+\cdots+1000=\frac{(1+1000)\times 1000}{2}=500,500$$

7. (a-i) $P_1 = P_0 \times r = 6 \times 1.5 = 9$.

(a-ii) In general, $P_N = P_0 \times r^N$, so we have $P_N = 6 \times (1.5)^N$.

(a-iii) $P_7 = 6 * 1.5^7 = 102.5156$.

(b) You're given the compound interest formula in the book:

$$P_N = P_0 \times \left(1+\frac{i}{k}\right)^{N\times k}.$$

In our situation, $N = 4$, $i = 0.08$, $k = 3$, and $P_0 = 500$. That leads to The compound interest formula in the book is:

$$P_4 = 500 \times \left(1+\frac{0.08}{3}\right)^{4\times 3} = \$685.68$$

8. (a) In the word "STREETS", there are two S's, two T's, two E's, and one R. Somehow we have to figure out how many different words we can arrange out of these letters; in other words, how many different ways can we fill up 7 blanks with these letters?

I like to do this problem with a box model. In the first box, I have 7 blanks, and I choose the two locations for the S's. In the second box, I have 5 remaining blanks, and I choose two locations for the T's. In the third box, with 3 blanks, I choose locations for the two E's, and then in the last box, I have one blank left over for my one R:

$$\boxed{{}_7C_2} \times \boxed{{}_5C_2} \times \boxed{{}_3C_2} \times \boxed{{}_1C_1} = ?$$

This all works out to:

$$\boxed{21} \times \boxed{10} \times \boxed{3} \times \boxed{1} = 630.$$

(b) As we discussed in class, this problem is exactly the same as: "Suppose you flip a coin 20 times. What is the probability that you get exactly 9 heads?"

The size of the sample space, N, is the total number of ways you can flip a coin 20 times. You can follow examples in your book to see $N = 2^20 = 1,048,576$.

We also need to know how many different ways you can get exactly 9 heads. I think of it like this: I have 20 blanks, and I need to write down "H" in exactly 9 of them. The order that I write them in doesn't matter, so there are

$$_{20}C_9 = \frac{20!}{(20-9)!\,9!} = 167,960$$

ways to do this. So the probability of getting exactly 9 heads is:

$$P(\text{exact 9 heads}) = \frac{\text{number of ways to get 9 heads}}{\text{total number of ways to flip a coin 20 times}} = \frac{167,960}{1,048,576} = \text{ about } 16.02\%$$

(c) The first dog digs three holes a month for 12 months, so he makes 36 holes total. The second dog digs three holes a month for 11 months, so she makes 33 holes total. The third dog makes 30 holes total, and so on, until you get to the very last dog. He's only around for one month, so he only makes 3 holes total. Hopefully you can see that the answer to this question is:

$$3 + 6 + 9 + \cdots + 30 + 33 + 36 = 234$$

(To get 234 I used the formula to sum an arithmetic sequence, which was given to you on the exam. That's how I intended for you to do the problem, too, but many people simply wrote out every number (3, 6, 9, 12, 15, and so on) and then added them together. I guess next time I'll have to make sure there are more than 12 numbers!)

A current version of these solutions is available at http://www.math.umn.edu/~rogness/math1001/

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