I haven't shown every detail in these solutions, although in most cases I've shown more work than you would need to get full credit. That makes these solutions look complicated. In truth, most of the problems (other than #2) can be done in 2-3 lines work.

If you're not sure about a step or don't understand why an answer is correct, talk to me or your TA.

1. (i) To find the number of *negative* real roots, we need to consider $f(-x)$. In this case, $f(-x) = -3x^9 + 4x^8 + 5x^7 + 3x^6 + 2x^3 + 3x^2 + 1$. This only has one sign change; therefore Descartes Rule of Signs says that $f(x)$ has one negative real root.

(ii) According to the Remainder Theorem, $f(3)$ is equal to the remainder after dividing $f(x)$ by $(x-3)$. That remainder is 1.

(ii) Neither of them are factors. You can do this problem a few different ways. The hard way is to do the division and see that your remainders are not zero. The easy way is to use the factor theorem, which says that $(x-c)$ is a factor of $f(x)$ if and only if $f(c) = 0$. In this case, $f(-1) \neq 0$ and $f(-2) \neq 0$, so neither $(x+1)$ or $(x+2)$ are factors.

(iv) Using the law of cosines, $c^2 = 4^2 + 3^2 - 2(4)(3)\cos 60° = 13$, so $c = \sqrt{13}$.

(v) You can do this with Heron's Formula if you wish, but that's the long way. [And if your answer to (iv) is incorrect you'll get this one wrong, too.] The easier way is this: Area = $(1/2)$(product of two sides)(sine of their included angle), i.e.

$$A = \frac{1}{2}(4)(3)(\sin 60°) = \frac{6\sqrt{3}}{2} = 3\sqrt{3}$$

2. I'll add this later - sorry! Most people did very well on this problem, so if you lost some points you can ask a friend in the class. I'll add the solution to this problem on Monday.

3. (i) You can do this by the "general method" given in the book and in lecture for this type of problem. I think it's easier to square this particular equation:

$$(\sin\theta - \cos\theta)^2 = 1^2$$
$$\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta = 1$$
$$-2\sin\theta\cos\theta + (\sin^2\theta + \cos^2\theta) = 1$$
$$-2\sin\theta\cos\theta + (1) = 1$$
$$-2\sin\theta\cos\theta = 0$$
$$-2\sin(2\theta) = 0$$
$$\sin(2\theta) = 0$$

This happens for $2\theta = ... - \pi, 0, \pi, 2\pi, 3\pi, 4\pi, ...$ or $\theta = ..., -\pi/2, 0, \pi/2, \pi, 3\pi/2, 2\pi$, so the angles in the correct range are $0, \pi/2, \pi, 3\pi/2$.

Remember, though – squaring the equation can introduce extraneous solutions. In this case, you can check that 0 and $3\pi/2$ turn out to be extraneous. So the answers are $\pi/2$ and π.

(ii) This is one of those quadratic equations. First, note that $\cos(2\theta) = 2\cos^2\theta - 1$, so we have:
$$2\cos^2\theta - 1 = 3\cos\theta - 2$$
$$2\cos^2\theta - 1 - 3\cos\theta + 2 = 0$$
$$2\cos^2\theta - 3\cos\theta + 1 = 0$$
$$(2\cos\theta - 1)(\cos\theta - 1) = 0$$
$$\cos\theta = 1/2 \text{ or } \cos\theta = 1$$

In the given interval, $\cos\theta = 1/2$ for $\theta = \pi/3, 5\pi/3$ and $\cos\theta = 1$ for $\theta = 0$.

4. To do this problem, you need to (a) follow the hint, and (b) remember that $\sin(2\theta) = 2\sin\theta\cos\theta$ and $\cos(2\theta) = \cos^2\theta - \sin^2\theta$. According to a trick used many times on the homework (and explained in detail on the formula study sheet), this means $\cos(6\theta) = \cos^2(3\theta) - \sin^2(3\theta)$, etc. You can do the work for this problem in a few lines. With *every* detail, it looks like this:

$$\begin{aligned}\cot(6\theta) &= \frac{\cos(6\theta)}{\sin(6\theta)} = \frac{\cos^2(3\theta) - \sin^2(3\theta)}{2\sin(3\theta)\cos(3\theta)} = \frac{\cos^2(3\theta)}{2\sin(3\theta)\cos(3\theta)} - \frac{\sin^2(3\theta)}{2\sin(3\theta)\cos(3\theta)} \\ &= \frac{\cos(3\theta)}{2\sin(3\theta)} - \frac{\sin(3\theta)}{2\cos(3\theta)} = \frac{1}{2}\frac{\cos(3\theta)}{\sin(3\theta)} - \frac{1}{2}\frac{\sin(3\theta}{\cos(3\theta)} \\ &= \frac{1}{2}\cot(3\theta) - \frac{1}{2}\tan(3\theta)\end{aligned}$$

5. Let $\alpha = \cos^{-1}\left(-1/\sqrt{2}\right)$ and $\beta = \tan^{-1}\left(-4/3\right)$. So we're looking for $\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$.

It's a little hard to write the solution to this problem without drawing pictures of α and β, so if you're confused talk to me or your TA. To draw α, put a $45° - 45° - 90°$ triangle in the second quadrant. To draw β, draw a 3-4-5 triangle in quadrant four. (The y-side is 4 units long, the x-side is 3 units long.) Once you draw the pictures correctly, you can read off the values that you need:

$$\begin{aligned}\sin(\alpha + \beta) &= \sin\alpha\cos\beta + \cos\alpha\sin\beta \\ &= \frac{1}{\sqrt{2}}\cdot\frac{3}{5} + \frac{-1}{\sqrt{2}}\cdot\frac{-4}{5} \\ &= \frac{3}{5\sqrt{2}} + \frac{4}{5\sqrt{2}} \\ &= \frac{7}{5\sqrt{2}}\end{aligned}$$