These are not meant to be detailed solutions. If you can't figure out why a given answer is correct, talk to your TA or me.

1. (i) C. Linear equations are of the form $A_1x_1+A_2x_2+\cdots+A_nx_n = C$, where the A ' s and C are constants and the x ' s are variables. So no exponents, square roots, or trig functions are allowed.

(ii) D. By DeMoivre's Theorem,

$$\begin{aligned} z^4 &= 3^4 \cdot \text{cis } (4 \cdot 45^\circ) \\ &= 81 \cdot \text{cis } (180^\circ) \\ &= 81 \cdot \cos(180^\circ) + 81i \cdot \sin(180^\circ) \\ &= 81(-1) + 0 = -81 \end{aligned}$$

(iii) A. Multpily both sides of the equation by r to get:

$$\begin{aligned} r^2 &= r\cos\theta - r\sin\theta \\ x^2+y^2 &= x-y \\ x^2+y^2-x+y &= 0 \end{aligned}$$

(vi) C. The ellipse is taller than it is wide, so the y-axis is the major axis. That means the foci are on the y-axis, which means C is the *only* possible answer, without even working out the numbers to find out the exact coordinates.

(v) D. The "general form" of a parabola which opens down is $x^2 = -4ay$. The distance from the vertex to the directrix is 1, so $a = 1$ by the (geometric) definition of a. Also, the parabola is centered at $(2, -1)$ which means x is replaced with $(x-2)$ and y is replaced with $(y+1)$. This all means that D is the only possible answer.

2. For $f(x) = 2x^4+3x^3+7x^2+9x+3$, the rational root theorem says that the only possible rational roots are $\pm1, \pm3, \pm1/2, and \pm3/2$. But we were told there are no positive real roots, so we can forget about the positive rational roots. (Remember, rational numbers are also real numbers.)

So we need to check the roots $-1, -3, -1/2, -3/2$. If you test -1, you find that it *is* a zero: $f(-1) = 0$. So we can factor out $(x-(-1)) = (x+1)$. Do the synthetic division (or long division of polynomials) and get:

$$f(x) = (x+1)(2x^3+x^2+6x+3)$$

There are two possible approaches now. You can keep testing rational roots, in which case you'd discover that $f(-1/2) = 0$ as well. Factoring again gives:

$$f(x) = (x+1)(x+1/2)(2x^2+6)$$

If you solve $2x^2+6 = 0$, you'll get $x = \pm\sqrt{3}\cdot i$.

Alternately, you could factor the depressed equation by grouping:

$$\begin{aligned} 2x^3+x^2+6x+3 &= x^2(2x+1)+3(2x+1) \\ &= (x^2+3)(2x+1) \end{aligned}$$

And find the roots $x = -1/2$, $x = \pm\sqrt{3} \cdot i$ from this.

3. Following the formula in the book, we have

$$z_k = \sqrt[3]{27} \cdot \text{cis } (10^\circ + 120^\circ k)$$

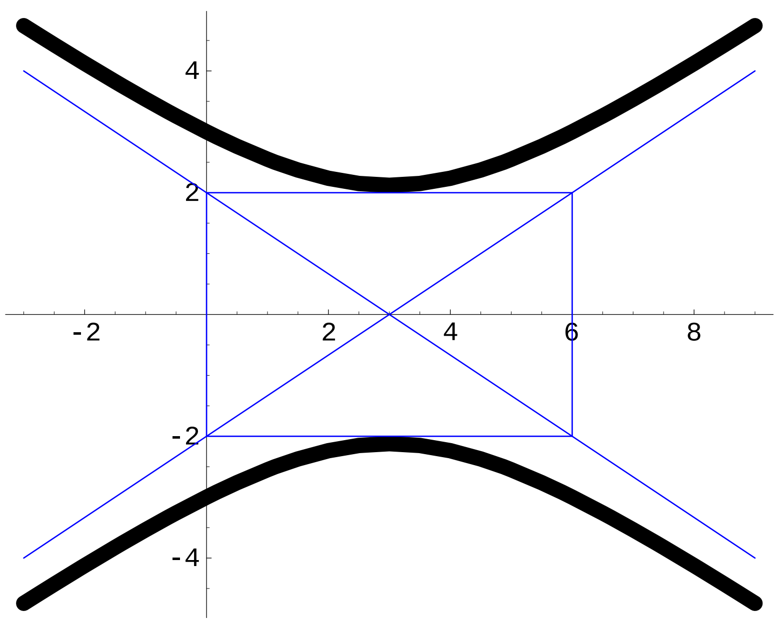
for $k = 0, 1, 2$. Plugging in those values of k gives

$$\begin{aligned} z_0 &= 3 \cdot \text{cis } (10^\circ) \\ z_1 &= 3 \cdot \text{cis } (130^\circ) \\ z_2 &= 3 \cdot \text{cis } (250^\circ) \end{aligned}$$

4. This is not a parabola, because there is an x^2 term *and* a y^2 term. They have different signs, so it's not an ellipse; it's a hyperbola. You need to begin by completing the square and converting the equation to the "standard" form:

$$\begin{aligned} -4x^2 + 24x + 9y^2 &= 72 \\ -4\left[x^2 - 6x + (-3)^2\right] + 9y^2 &= 72 + -4 \cdot (-3)^2 = 72 - 36 = 36 \\ -4(x-3)^2 + 9y^2 &= 36 \\ \frac{-4(x-3)^2}{36} + \frac{9y^2}{36} &= 1 \\ \frac{y^2}{4} - \frac{(x-3)^2}{9} &= 1 \end{aligned}$$

This is a hyperbola, centered at (3,0). The y^2 term is positive, so it opens up and down. If you draw the box, lines, and hyperbola it looks something like this:



5. Here's a rough sketch of one possible way to solve the problem. This is what we start with:

$$\begin{array}{cccccc} x & -y & +z & = & 2 \\ 2x & & -2z & = & 4 \\ -3x & +5y & -4z & = & -3 \end{array}$$

Now we can replace equation (2) with $(2) - 2(1)$:

$$\begin{array}{rrrcr} x & -y & +z & = & 2 \\ & +2y & -4z & = & 0 \\ -3x & +5y & -4z & = & -3 \end{array}$$

And now replace (3) with $(3) + 3(1)$:

$$\begin{array}{rrrcr} x & -y & +z & = & 2 \\ & +2y & -4z & = & 0 \\ & +2y & -z & = & 3 \end{array}$$

Now replace (3) with $(3) - (2)$

$$\begin{array}{rrrcr} x & -y & +z & = & 2 \\ & +2y & -4z & = & 0 \\ & & 3z & = & 3 \end{array}$$

This immediately leads to $z = 1$, and you can back-substitute to find $y = 2$ and $x = 3$.