Exam 2 covers Section 7 through Section 16, with the following caveats:

- Sections 9 and 15 are not part of our course.
- The only thing you need to know from Section 14 is the following characterization from the Heine-Borel Theorem: *a subset $S$ of $\mathbb{R}$ is compact if and only if it is closed and bounded.* Thus any questions about compact sets will actually be questions about sets which are closed and/or bounded.
- In Section 11, the ordered field $\mathbb{F}$ is not a central idea for our class and will not appear on this test. You are also not required to memorize the 15 axioms in this section.

As you may recall, the solutions to the exams from the Spring 2009 semester are online. I have included relevant questions from those exams here to give you a sample of possible test questions. Because the course used a different textbook, material was covered in a slightly different order, so these questions are on both the first and second exams from that semester:

**Spring 2009 Exam 1.** (Solutions at http://www.math.umn.edu/~keynes/3283Exam1Solutions.pdf.)

4. Use induction to prove $5^n + 6^n < 7^n$ for all $n > N$, for some $N$. (Hint: this fails for $n = 1$, for example, so you must determine the lowest value for which the statement holds, and use that as the basis for your induction.)

5(a). Let $A = \left\{ \frac{n-1}{n+1} \,|\, n \in \mathbb{N} \right\}$.

   i. Show that $A$ is bounded. (i.e. bounded above *and* below)

   ii. Find $L = \inf A$ and $M = \sup A$ (no proof needed for this part). Determine whether or not $L \in A$ and $M \in A$.

   iii. Prove the value $M = \sup A$ you found above is in fact the least upper bound of $A$.

5(b). Determine if the following sets are bounded above or below. In each case, if the set is bounded above, find the supremum. If the set is bounded below, find the infimum.

   i. $\left\{3 + \frac{1}{2}, -2 + \frac{1}{2}, 3 + \frac{1}{4}, -2 + \frac{1}{4}, 3 + \frac{1}{8}, -2 + \frac{1}{8}\right\}$.

   ii. $\left\{x \in \mathbb{R} \,|\, x > 0 \text{ and } x^2 - 4x + 3 > 0\right\}$

   iii. $\left\{x \in \mathbb{R} \,|\, x^3 - x < 0\right\}$

   iv. $\{1 - .3, 2 - .33, 3 - .333, 4 - .3333, 5 - .33333\} \cup \left\{ \frac{1}{\sqrt{n}} \,|\, n \in \mathbb{N} \right\}$

**Spring 2009 Exam 2.** (Solutions at http://www.math.umn.edu/~keynes/3283Exam2Solutions.PDF.)

1. Prove that $a_n \to 0$ if and only if $|a_n| \to 0$.
3. Assume $a_n \to L$.
   (a) Carefully and precisely write the definition of $a^n \to L$.
   (b) Prove that if $L < 0$ there exists $n \in \mathbb{N}$ such that $a_n < 0$.
   (c) Prove in one line: if for every $n \in \mathbb{N}$, $a_n \geq 0$ then $L \geq 0$.

4(c). (I have changed this problem slightly; use Theorem 16.8!) Prove $\dfrac{\sin^2(n^n)}{n+1} \to 0$.

These problems give a good sampling of the material in Section 10, 12 (although density of the rationals was not used above) and 16. For the other Sections – mainly 7, 8, and 13 – the homework problems give an excellent idea of what we consider to be the important concepts and problems.

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