Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 12/1

HOMEWORK ASSIGNMENT

Regular Problems:

(1) The following pictures are representations of infinite series. Certain portions of the squares are shaded according to some infinite process; the pictures only show the first few stages, represented by different colors. Determine the infinite series represented by each picture and then find the sum of the series by determining the overall portion of the square which would be shaded at the end of the infinite process.
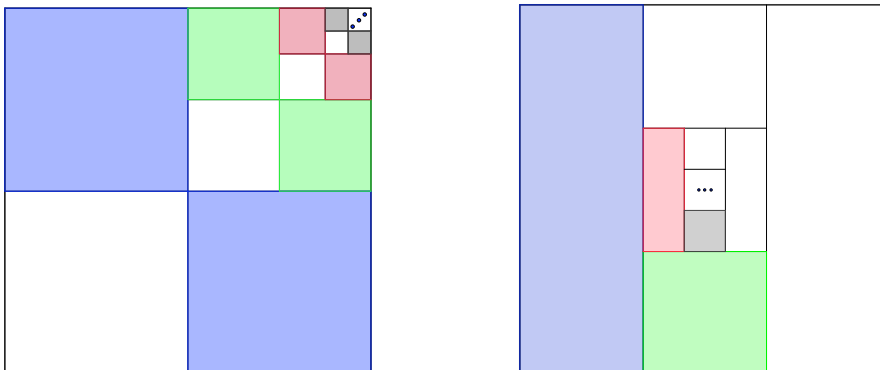
(2) Suppose $\sum a_n = s$ and $\sum b_n = t$. Prove $\sum (a_n - b_n) = s - t$ using definitions and theorems from §17. (Do not use Theorem 32.4; you are being asked to prove a variant of it.)

For this assignment we have the following tools to prove the convergence of a series: direct analysis of the partial sums (as with telescoping series or the formula for a geometric series), the comparison test, the alternating series test, and the theorem that if $\sum |a_n|$ converges, so does $\sum a_n$. We also have the contrapositive of Theorem 32.5: if $\lim a_n \neq 0$, then $\sum a_n$ diverges. Using just these tools, determine whether the following series **converge absolutely**, **converge conditionally**, or **diverge**. In each case the answer alone is worth little no no credit; you must explain which theorem or technique you are using and show how it applies. If you use the comparison test, you should compare to a series whose behavior was proven in class or earlier on this assignment, or else prove yourself whether it converges or diverges.

(3) $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$ (I stated this series converges, but we haven't yet proven it.)

(4) $\sum_{n=1}^{\infty} \dfrac{(-1)^n \, n}{n+1}$

(5) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{2n^2 + n + 1}$

(6) $\displaystyle\sum_{n=1}^{\infty} \frac{\cos(n\pi)}{n}$

(7) $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n(n+1)}$

Writing Problem 1: Given $\sum a_n$ and $\sum b_n$, suppose there exists an N such that $a_n = b_n$ for all $n > N$. Prove that $\sum a_n$ is convergent if and only if $\sum b_n$ is convergent. Explain why this means the convergence of a series is not affected by changed a finite number of terms in the sum.

In lieu of a second writing problem, your assignment this week includes an extra credit problem, worth up to five writing points:

Extra Credit Problem (5 Points) Write a proof of the ratio test as stated in class:

Theorem (Ratio Test). *Let $\sum a_n$ be a series of nonzero terms.*

- *If* $\lim \left| \dfrac{a_{n+1}}{a_n} \right| < 1$, *then* $\sum a_n$ *converges absolutely.*
- *If* $\lim \left| \dfrac{a_{n+1}}{a_n} \right| > 1$, *then* $\sum a_n$ *diverges*

As stated in your book, the Ratio Test involves $\lim\sup$ and $\lim\inf$, which are defined in a section that we didn't cover. Hence the book's version (and its proof) contain terms and reasoning which don't apply for our course. However, the proofs that you find in other books (or on Wikipedia) are not detailed enough for our course. You are welcome to read those proofs, but you should write out your own solution without referring to another book, student, or other resources. The level of detail as presented in class on Wednesday, 11/24, would be appropriate.

This problem should be handed in with the rest of the assignment on Thursday, 12/1. Because it is for extra credit, no rewrites of this problem will be allowed.