Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 10/20

Special Note: As mentioned in class, I've presented open and closed sets in "reverse order" as compared to the book. The topology problems here can be solved using neighborhoods, interior points, definition of open set as a set whose points are all interior points, and a closed set is one whose complement is open. Your solutions should not use boundary points, accumulation points or closures. Those will appear on the next assignment.

Some of the proofs involving open and closed sets on this assignment are widely available in your book and other sources, but they are important enough that I am still asking you to write them out to help you learn the concepts. Because they are so widely available, it is even more important that you write the proofs **in your own words**. In particular, you should be able to write your proof down without any other resource in front of you.

Homework Assignment

Regular Problems:

(1) Identify the minimum, maximum, infimum and supremum (if they exist) of each of the following sets. Prove that your answers satisfy the definitions in 12.2, 12.5 and 12.6.
  - (a) $A = \left\{ \frac{(-1)^n}{n} \,\middle|\, n \in \mathbb{N} \right\}$
  - (b) $B = \left\{ 1 - x^2 \,|\, x \in \mathbb{R} \right\}$
  - (c) $C = \left\{ 1 - \frac{1}{n^2} \,|\, n \in \mathbb{N} \right\}$
  - (d) $D = \mathbb{Q} \cap [1, 5] = \{q \in \mathbb{Q} \,|\, 1 \leq q \leq 5\}$

(2) Let S be a nonempty bounded subset of $\mathbb{R}$ and let $m = \inf S$. Prove $m \in S$ if and only if $m = \min S$.

(3) Prove: if x and y are real numbers with $x < y$, then there are infinitely many *irrational* numbers in the interval $[x, y]$. (Hint: assume not, in which case you have a set of finitely many irrational numbers. What would Problem **??** be like for any finite set? How could that lead to a contradiction?)

(4) Given a set $S \subseteq \mathbb{R}$, we define the interior of S, written int S, to be the set of all interior points of S. Find the interior of each set in Problem **??**.

(5)
  - (a) Prove that any intersection of finitely many open sets is open.
  - (b) Determine the set $S = \bigcap_{n=1}^{\infty} \left(-\frac{1}{n}, 2 + \frac{1}{n}\right)$ and prove your answer is correct. Why is the word "finitely" important in Part **??**?

(6) Determine the set $T = \bigcup_{n=1}^{\infty} \left[-1 + \frac{1}{n}, 1 - \frac{1}{n}\right]$ and prove your answer is correct. Does this contradict what you are asked to prove in Writing Problem 2?

Writing Problem 1: Suppose A and B are two bounded subsets of the reals. Prove: if $a \leq b$ for all $a \in A$, $b \in B$, then $\sup A \leq \sup B$.

Writing Problem 2: Using the definitions (and related theorems) for open and closed sets from class, prove:
  - (a) The intersection of any (finite or infinite) collection of closed sets is closed.
  - (b) The union of finitely many closed sets is closed.