Implicit in any homework problem is that you must explain why your answer is correct, even if the problem does not ask for a formal proof. Writing problems should have complete explanations of your work, written in complete sentences with correct grammar.

Due: Thursday, 11/17

Homework Assignment

(As a reminder, since we took a break from homework while preparing for the second exam: this homework covers Section 18 and the beginning of Section 32.)

Regular Problems:

(1) Prove: $s_n = ((-1)^n\, n)$ is an unbounded sequence which does not diverge to $+\infty$ or $-\infty$. (Hint: there are three things to prove; make sure you explain why each of them is true.)

(2) Use the Monotone Convergence Theorem to show the following sequences converge.
- (a) $a_n = \dfrac{n^2-1}{n(n+1)}$
- (b) $b_n = e^{-n}$. (You may use your prior knowledge of the function $f(x) = e^x$.

(3) Prove the sequence $c_n = \frac{1}{n^2}$ is a Cauchy sequence.

(4) For each series $\sum a_n$, find an expression for the partial sum $s_n = a_1 + a_2 + \cdots + a_n$. Then find the sum of the series or show it is divergent. (In each case you are expected to show supporting work for your answer.)
- (a) $\displaystyle\sum_{n=1}^{\infty} \frac{3}{(3n+2)(3n-1)}$
- (b) $\displaystyle\sum_{n=1}^{\infty} \frac{(n-1)!}{(n+1)!}$

Writing Problem 1: Prove that the following sequence converges and find its limit.

$$b_1 = 1, \quad b_{n+1} = \sqrt{12 + b_n}$$

Writing Problem 2:

(a) Use induction to prove $1 + r + r^2 + \cdots + r^n = \dfrac{1-r^{n+1}}{1-r}$ for $r \neq 1$.

(b) For $|r| < 1$, prove carefully (using the sequence of partial sums and the limit laws in Section 17) that

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$