The final exam is scheduled for Saturday, 17 December 2011, from 4-6pm. Sections 11 and 12, which meet at 10:10am on Tuesday/Thursday with Nick Switala and Ji Hee Kim, will take the exam in Murphy 130. Sections 13, 14 and 15, which meet at 11:15am on Tuesday/Thursday with Liping Li, Hui Li and Erin Manlove, will take the exam in Rapson 100.

At 120 minutes, the final is roughly 2.5 times the length of our 50 minute midterms. It covers sections 1-8, 10-14, 16-18 and 32-34, but the caveats from Exam 2 still apply; you should re-read the study guide for that exam to see what material from Chapter 3 you don't have to worry about:

http://www.math.umn.edu/~rogness/math3283w/3283-f11-exam2-review.pdf

Other than questions about Section 34, Power Series, the rest of the problems will be more or less evenly distributed amongst the topics from the earlier midterms.

Roughly speaking, there are three types of problems on the exam:

**Proofs:** Each of the midterms included a longer proof, and you will be asked to write a few on the final exam. As a reminder, this could include:

- Set theory style proofs, similar to Theorem 5.13.
- Topological proofs about open and closed sets, such as these four related results:

  *The intersection of two open sets is open.*
  *The union of two open sets is open.*
  *The intersection of two closed sets is closed.*
  *The union of two closed sets is closed.*

  (Also: which of these statements is true when *two* is replaced by *infinitely many*?)
- Proofs which are intended to check your knowledge of the different methods of proof: direct proof, contrapositive, proof by contradiction, and induction. On the first midterm, for example, you had to use two different methods to prove facts about integers. On the second midterm you were asked to prove the generalized triangle inequality by induction. (This was one of two proofs by induction which appeared as writing problems throughout the semester.)

**Short Proofs / Applying Definitions or Theorems:** Some problems will ask you to prove something which amounts to verifying definitions or applying some other theorem, as opposed to coming up with your own argument for why something is true. These might be more computational in nature, but you still need to justify your reasoning. Examples of these problems include:

- Proving that a relation is an equivalence relation.
- Proving that a given function is an injection / surjection / bijection.
- Proving that a certain number is an upper/lower bound (or supremum or infimum) of a set $S$.
- Proving that a given sequence converges according to the definition, or by using a tool such as Theorem 16.8 or Theorem 17.1).
- Proving that a given series converges according to the definition or by using a convergence test.
- Proving that a given power series has a certain interval of convergence.

Many of these have potential followup questions; for example, after proving a relation is an equivalence relation, on Exam 1 you were asked to describe the equivalence class of a certain element.

**Short Answer:** As on the last two midterms, you can expect some short answer problems where you give your answer without a full-blown proof, although you might still be asked to justify your solution by citing a theorem or definition.

## How to Study for the Test

As mentioned on previous study guides, you should learn the definitions and theorems, but there is no substitute for *doing problems*. You can understand the comparison test, but if you haven't done a lot of comparison problems, you may not realize what to compare $\sum \frac{1}{3+2^n}$ to. You might understand the idea of an injective function, but if you haven't constructed injective functions between various sets, you will be hard pressed to do so on the exam.

You can look at your lecture notes, homework assignments, and the midterm exams to get a good feel for what topics I feel are important and are likely to appear on the test. Redo any homework and exam problems that you struggled with. The TAs and I can help you with these during office hours and class sessions. If you're stuck, make use of your lecture notes, textbook and other resources, like us. Once you figure out a problem, pick other similar exercises in your textbook and work on those, with the goal of being able to solve them confidently without referring to your notes or other resources.

An important technique for studying definitions and theorems is to come up with your own examples to learn why certain distinctions and conditions are important. Can you quickly give an example of a series which converges conditionally but not absolutely? Can you give an example of two infinite sets which are not equinumerous? In Theorem 16.8, why is it important that $\lim a_n = 0$? Can you find a counter-example to show that removing this condition will make the theorem false?

## Suggested Problems

In addition to exercises from section 34, I've written suggested problems below for a few sections that many people found difficult. You can also look at the review problems suggested for the first three exams.

**Section 34:** 34.1, 34.2, enough parts of 34.3 and 34.5 that you are comfortable with the techniques, and 34.7.

**Section 7:** The notation on page 65 is important, as are some of the theorems on the following page, but first and foremost you should make sure you are comfortable with injections, surjections and bijections. The following problem has 18 parts, and I do not expect you to do them all, but you should do enough of them that you would be able to answer any similar problem.

- Consider the sets $A = \{1, 2, 3, \ldots, 10\}$, $\mathbb{N}$ and $\mathbb{R}$. Find injections (or explain why none is possible) $A \to \mathbb{N}$, $A \to \mathbb{R}$, $\mathbb{N} \to \mathbb{R}$, $\mathbb{N} \to A$, $\mathbb{R} \to A$, $\mathbb{R} \to \mathbb{N}$. Replace "injections" with "surjections" and then "bijections" and repeat the problem.

**Section 8:** 8.1, 8.2, 8.3(d,e), 8.10. Also:

- Prove: if $S$ is denumerable, then adding one element to $S$ does not change its cardinality. Is the same true if $S$ is uncountable?

**Section 12:** 12.1, 12.2, 12.3

**Section 13:** The problems mentioned in the **Proofs** section above are most important; exercises 13.1-13.5 can help you review some of the definitions.