The following is a non-comprehensive list of solutions to the skills problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

16.6(a) There's are "thinking" and "proof" portions of this problem.

Thinking: For a given $\epsilon > 0$ we want to figure out how large n must be to ensure $|k/n - 0| < \epsilon$. Using algebra, this last inequality is equivalent to $|k|/\epsilon < n$. So we set $N = |k|/\epsilon$ in the next portion.

Proof: Let $\epsilon > 0$ be given, and set $N = |k|/\epsilon$. Then for all $n > N$ we have

$$|k|/\epsilon < n \Leftrightarrow |k|/n < \epsilon$$

which means

$$|k/n - 0| = |k/n| = |k|/n < \epsilon.$$

Hence $\lim k/n = 0$, as desired.

16.6(b) Without writing out the formal proofs, there are two cases to consider.

- If $k \geq 1$, then $n^k \geq n$ for all $n \in \mathbb{N}$; hence $1/n^k \leq 1/n$ for all n, and we can apply Theorem 16.8 with $a_n = 1/n$ on the right hand side. (In Theorem 16.8 the constant k–which is different than the exponent in $1/n^k$–would be $k = 1$.

- If $0 < k < 1$, things are a little trickier, because then $n^k < n$ and $\dfrac{1}{n} < \dfrac{1}{n^k}$, so we can't use Theorem 16.8 with $a_n = 1/n$. Hence we go back to the definition! Given $\epsilon > 0$, how large does n have to be to ensure that
$$\left|\frac{1}{n^k} - 0\right| = \frac{1}{n^k} < \epsilon?$$
Using algebra, we find $n > \left(\dfrac{1}{\epsilon}\right)^{1/k}$, so set $N = \left(\dfrac{1}{\epsilon}\right)^{1/k}$ and proceed with the formal proof, like in 16.6(a).

16.10 There are many possible correct answers for each part of this problem.

(a) One possible answer would be $a_n = \left(1 + \dfrac{1}{n}\right)^n$, which converges to e. (You don't need to prove that convergence in this particular problem; you might recall that this is actually the definition of e in some precalculus or calculus textbooks.) Each number in this sequences is a power of a rational number, and hence is rational. Its limit, e, is not rational number.

(b) One possible answer would be $b_n = \dfrac{\sqrt{2}}{n}$. Then each b_n is irrational, but $b_n \to 0 \in \mathbb{Q}$.