The following is a non-comprehensive list of solutions to the skills problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

(8.4) Many functions $f : (0,1) \to (m,n)$ are possible here. A linear function like $f(x) = (n-m)x + m$ might be the simplest. Ask us if you're not clear why this function is a bijection or maps one interval on to the other.

(8.10) If S is denumerable, then (as shown in class) we can write the elements of S out as a list:[1]

$$S = \{s_1, s_2, s_3, \ldots\}$$

But then $\mathbb{N}$ is also equinumerous with

$$T = \{s_2, s_4, s_6, s_8, \ldots\}$$

which is a proper subset of S. Hence $|S| = |\mathbb{N}| = |T|$ and, in particular, $|S| = |T|$.

Note: this problem is generalizing the example in class that the set of integers $\mathbb{Z}$ is equinumerous with $2\mathbb{Z}$, the set of *even* integers.

(10.6) We first anchor this statement at $n = 1$: $\frac{1}{1(1+1)} = \frac{1}{2}$, as desired. For the induction step, assume the statement holds for $n = k$ and prove that it holds for $n = k+1$. We start by writing out the left hand side for $n = k+1$ and (using what we know about the first k terms) manipulate it into the desired formula:

$$\begin{aligned}
\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k(k+1)} + \frac{1}{(k+1)(k+2)} &= \left(\frac{1}{1\cdot 2} + \frac{1}{2\cdot 3} + \cdots + \frac{1}{k(k+1)}\right) + \frac{1}{(k+1)(k+2)} \\
&= \left(\frac{k}{(k+1)}\right) + \frac{1}{(k+1)(k+2)} \text{ (by assumption)} \\
&= \frac{k(k+2)+1}{k(k+1)(k+2)} \\
&= \frac{k^2+2k+1}{k(k+1)(k+2)} \\
&= \frac{(k+1)^2}{(k+1)(k+2)} \\
&= \frac{(k+1)}{(k+2)}
\end{aligned}$$

Which is exactly the formula we want, for $n = k+1$.

(10.7) Again, anchor the statement when $n = 1$ by checkin that $1 + r$ is in fact equal to $\dfrac{1-r^2}{1-r}$. (Hint: factor the top as a difference of squares and cancel; why is it important that $r \neq 1$ here?)

[1] That's because there exists a bijection $f : \mathbb{N} \to S$, so the elements of S are $\{f(1), f(2), f(3), \ldots\}$. To make the notation a little cleaner, we're defining $s_n = f(n)$.

Now assume the statement is true for $n = k$ and prove it for $n = k + 1$:

$$\begin{aligned}
1 + r + r^2 + \cdots + r^n + r^{n+1} &= \left(1 + r + r^2 + \cdots + r^n\right) + r^{n+1} \\
&= \left(\frac{1 - r^{n+1}}{1 - r}\right) + r^{n+1} \text{ (by assumption)} \\
&= \frac{1 - r^{n+1} - r^{n+1}(1 - r)}{1 - r} \\
&= \frac{1 - r^{n+1} + r^{n+1} - r^{n+2}}{1 - r} \\
&= \frac{1 - r^{n+2}}{1 - r}
\end{aligned}$$

Which is the formula we want to prove, for $n = k + 1$.

(10.15) (a) This is not a correct application of using $P(k)$ to prove $P(k + 1)$. I'd suggest trying to do this proof physically with two marbles of different colors, say red and blue. If you remove the red one, you leave a set of one color (blue). If you then remove the other one, you leave a set of one color (red). But that does not imply that all of them must be the same color.

(b) There is no anchoring here; the statement is never verified for a specific value of n.