Quiz 1 . Thursday September 13, 2012

① Verify the tautology $[p \vee (q \vee r)] \iff [(p \vee q) \vee r]$.

p	q	r	$p \vee q$	$q \vee r$	$p \vee (q \vee r)$	$(p \vee q) \vee r$
T	T	T	T	T	T	T
T	T	F	T	T	T	T
T	F	T	T	T	T	T
T	F	F	T	F	T	T
F	T	T	T	T	T	T
F	T	F	T	T	T	T
F	F	T	F	T	T	T
F	F	F	F	F	F	F

The columns for $p \vee (q \vee r)$ and $(p \vee q) \vee r$ are identical, meaning that these statements have the same truth value for all possible triples of truth values for p, q, and r. Thus, the two statements are logically equivalent.

② A function f is strictly increasing $\iff$

$$\forall x, y, \quad x < y \implies f(x) < f(y).$$

Negation: $\exists\, x, y \ni x < y \wedge f(x) \geq f(y)$.

Math 3283W Fall 2012 - Writing Quiz #1
Thursday 9/13/12 - 20 minutes **Name:** Solutions

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (7 Points) Use a truth table to verify the following tautology: $[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$. *Make sure to explain why your table proves the desired result.*

To say this statement is a tautology is to say it is true for all possible values of True or False assigned to the "atomic" statements p, q, r. Thus, the unblemished column of "T"s in the table below verifies the tautology.

p	q	r	$(q \vee r)$	$p \vee (q \vee r)$	$(p \vee q)$	$(p \vee q) \vee r$	$[p \vee (q \vee r)] \Leftrightarrow [(p \vee q) \vee r]$
T	T	T	T	T	T	T	T
T	T	F	T	T	T	T	T
T	F	T	T	T	T	T	T
F	T	T	T	T	T	T	T
T	F	F	F	T	T	T	T
F	T	F	T	T	T	T	T
F	F	T	T	T	F	T	T
F	F	F	F	F	F	F	T

(2) (7 Points) Rewrite the following statement using logical symbols such as (but not limited to) $\forall$, $\exists$, $\ni$ and $\Rightarrow$ as appropriate. Then write the negation of the statement, to explain when a function is not strictly increasing, using the same symbolism.

A function f is *strictly increasing* iff for every x and for every y, if $x < y$, then $f(x) < f(y)$.

Symbolic version: $\forall x \, \forall y \left((x < y) \Rightarrow (f(x) < f(y)) \right)$

Negation (in steps): $\sim \left(\forall x \, \forall y \left((x<y) \Rightarrow (f(x) < f(y)) \right) \right)$

$\Leftrightarrow \quad \exists x \ni \sim \left(\forall y \left((x<y) \Rightarrow (f(x) < f(y)) \right) \right)$

$\Leftrightarrow \quad \exists x \ni \exists y \ni \sim \left((x<y) \Rightarrow (f(x) < f(y)) \right)$

$\Leftrightarrow \quad \exists x \, \exists y \ni \left((x<y) \wedge \sim (f(x) < f(y)) \right)$ (two "such that"s are superfluous)

$\Leftrightarrow \quad \boxed{\exists x \, \exists y \ni (x<y) \wedge (f(x) \geq f(y))}$