Math 3283W Fall 2012 - Writing Quiz #3

Thursday 9/27/12 - 20 minutes **Name:** Joe Fourier

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (7 Points) Let A, B, and C be subsets of a universal set U. Prove or give a counterexample: $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$.

We claim that this statement is true.

Proof: To show that $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$, take some $x \in A \setminus (B \cup C)$. Then, by definition, $x \in A$ and $x \notin B \cup C$. $x \notin B \cup C$ is equivalent to $x \notin B$ and $x \notin C$ $[\neg(x \in B \vee x \in C) \iff x \notin B \wedge x \notin C]$. So x lies in both $A \setminus B$ and $A \setminus C$; so $x \in (A \setminus B) \cap (A \setminus C)$, which shows that $A \setminus (B \cup C) \subseteq (A \setminus B) \cap (A \setminus C)$.

Conversely, suppose $x \in (A \setminus B) \cap (A \setminus C)$. Then x lies in both $A \setminus B$ and $A \setminus C$. So $x \in A$, $x \notin B$, and $x \notin C$. The latter two statements are equivalent to saying $x \notin B \cup C$. So $x \in A \setminus (B \cup C)$, which shows that $(A \setminus B) \cap (A \setminus C) \subseteq A \setminus (B \cup C)$. So the two sets are equal.

(2) (7 Points) Determine $\displaystyle\bigcap_{B \in \mathcal{B}} B$ for $\mathcal{B} = \left\{ \left[1, 1 + \frac{1}{n}\right] : n \in \mathbb{N} \right\}$ and prove that your answer is correct.

We claim that $\displaystyle\bigcap_{B \in \mathcal{B}} B = \{1\}$.

Proof: Indeed, $\{1\} \subseteq \bigcap_{B \in \mathcal{B}} B$ since $1 \in B = [1, 1 + \frac{1}{n}]$ for every $B \in \mathcal{B}$. Now consider $x \in \bigcap_{B \in \mathcal{B}} B$. Then $x \in B$ for all $B \in \mathcal{B}$, which is the same saying that $1 \leq x \leq 1 + \frac{1}{n}$ for every $n \in \mathbb{N}$. It cannot be the case that $1 < x$: if it were, we could take $N \in \mathbb{N}$ such that $1 + \frac{1}{N} < x$, which would contradict our assumption that $x \in B$ for *every* $B \in \mathcal{B}$. Thus $1 \geq x$. Combining this with our assumption that $1 \leq x$, we have that $x = 1$. So $\bigcap_{B \in \mathcal{B}} B \subseteq \{1\}$, giving us that $\bigcap_{B \in \mathcal{B}} B = \{1\}$.