**Math 3283W Fall 2012 - Writing Quiz #6**
**Thursday 11/1/12 - 20 minutes** **Name:** Key

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) Using the definitions from lecture, prove that the intersection of infinitely many closed sets is ~~an~~ a closed ~~open~~ set. (If you reduce/modify this statement to one about open sets, then you should prove that statement about open sets and not just cite a previous result.)

Let $\{A_n \mid n \in \mathbb{N}\}$ be a family of closed sets. Then $\forall n \in \mathbb{N}$, $B_n = \mathbb{R} \setminus A_n$ is an open set (by def'n of "closed").
To prove that $\bigcap_{n=1}^{\infty} A_n$ is closed, we need to show that $\mathbb{R} \setminus \bigcap_{n=1}^{\infty} A_n$ is open.
By DeMorgan's law, $\mathbb{R} \setminus \bigcap_{n=1}^{\infty} A_n = \bigcup_{n=1}^{\infty} B_n$. So let $x \in \bigcup_{n=1}^{\infty} B_n$.
Then $x \in B_K$ for some $K \in \mathbb{N}$, and since $B_K$ is open, there is some neighborhood $N(x, \varepsilon) \subset B_K \subset \bigcup_{n=1}^{\infty} B_n$. Thus $x$ is an interior point of $\bigcup_{n=1}^{\infty} B_n$, so $\bigcup_{n=1}^{\infty} B_n$ is open. $\square$

(2) Determine whether the set $S = \left\{1 - \frac{1}{n} : n \in \mathbb{N}\right\} \subset \mathbb{R}$ is open, closed, both or neither. Justify your answer using any definitions or theorems from class.

Neither. To see that $S$ is not open, we show $0 \in S$ is not an interior point of $S$:
$0 \in S$ because $0 = 1 - \frac{1}{1}$. But any neighborhood $N(0, \varepsilon)$ contains negative numbers, and $1 - \frac{1}{n} > 0 \ \forall n \in \mathbb{N}$.
To see that $S$ is not closed, we show $1 \in \mathbb{R} \setminus S$ is not an interior point of $\mathbb{R} \setminus S$:
$1 \in \mathbb{R} \setminus S$ because $\frac{1}{n} > 0 \ \forall n \in \mathbb{N}$, so $1 - \frac{1}{n} < 1 \ \forall n \in \mathbb{N}$.
But for any neighborhood $N(1, \varepsilon)$, by the Archimedean property, $\exists N \in \mathbb{N}$ s.t. $N > \frac{1}{\varepsilon}$. So $\varepsilon > \frac{1}{N}$, so $1 - \varepsilon < 1 - \frac{1}{N} < 1$.
Thus $1 - \frac{1}{N} \in N(1, \varepsilon)$, so $N(1, \varepsilon) \not\subseteq \mathbb{R} \setminus S$. $\square$