§1 Logical Connectives

Math consists of statements, sentences which can be classified as true/false — although we might not know which!

Ex Which are statements?

p: $2+2=4$ yes - true!

q: $3+3=10$ yes - false!

r: this statement is false No.

s: It's cold outside yes - assuming cold is defined

t: Truth is beauty. No.

u: $x^2-4x+3=0$. Yes - truth value depends on value of x

Given statements p, q we can create new ones using logic operators.
sentential connectives.

① Negation ($\neg$, $\sim$)

$\neg p$ is true when p is false, false when p is true.

Can represent w/ "Truth Table":

p	$\neg p$
T	F
F	T

② Conjunction ($\wedge$, and)

$p \wedge q$ is true when **both** p and q are true, otherwise it's false.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

3 Disjunction ($\vee$, or)

$p \vee q$ true if p is true, q is true or both

p	q	$p \wedge q$
T	T	T
T	F	T
F	T	T
F	F	F

⚠ Rarer: Exclusive or ($\underline{\vee}$, XOR)

p xor q true if p or q is T but NOT both.

Ex p = Jim is tall
q = Jim has red hair

$p \wedge q$ = Jim is tall and has red hair.

$\neg(p \wedge q)$ = NOT (Jim is tall and has red hair)

= Jim is not tall or Jim doesn't have red hair.

= Jim is not tall or doesn't have red hair.

don't write $\neg p \wedge q$; that's $(\neg p) \wedge q$

<u>You try</u>: truth table for $(\sim p) \vee (\sim q)$

⚠ $\sim(p \wedge q)$ is T/F precisely when $(\sim p) \vee (\sim q)$ is T/F. We say these statements are <u>logically equivalent</u>. This is one of De Morgan's Laws:

$$\sim(p \wedge q) \equiv (\sim p) \vee (\sim q)$$

In Words:

(4) Implications ($\Rightarrow$, if ..., then ...)

If p, then q. $\boxed{p \Rightarrow q}$

p = antecedent (hyp.)
q = consequent (conclusion)

Mathematicians use following convention: $p \Rightarrow q$ false only if p true and q is false. otherwise it's true.

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

<u>Ex</u> Determine truth values:

(T) If $\underbrace{\text{2 is positive}}_{T}$, then $\underbrace{\text{4 is even}}_{T}$.

(F) If $\underbrace{\text{3 is odd}}_{T}$, then $\underbrace{\text{pigs can fly}}_{F}$.

(T) If $\underbrace{\text{pigs can fly}}_{F}$, then $\underbrace{\text{I'm a rock star}}_{F/T}$.

If $p \Rightarrow q$ is true,
and $q \Rightarrow p$ is true, we write

$$\boxed{p \Longleftrightarrow q}$$

This is shorthand for "p and q are logically equivalent:

p	q	$p \Rightarrow q$	$q \Rightarrow p$	$\overset{(p\Rightarrow q)\wedge(q\Rightarrow p)}{\overset{\shortparallel}{p \Longleftrightarrow q}}$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

We also write: