§2. Quantifiers

"Quantifiers".... quantify things. In logic we mostly care if a statement is true in all cases, [at least] one case, or never.

$\boxed{\text{Being lazy, we use symbols}}$

Existential Quantifiers

$\exists$: there exists... (at least one

$\exists!$: there exists a <u>unique</u> (exactly one)

$\nexists$: there does not exist

Universal

$\forall$: for all

Other notation

$\ni$: such that

$p(x)$: stmt whose truth value depends on value of x.

$p(x): x^2-1=0$ $\quad p(0)$ false
$p(1)$ true.

Ex Write these stmts in symbols:

For some x, $x^2-1=0$.

$\exists x \ni x^2-1=0.$

For every real number $x>0$, there is a y s.t. $y^2=x$. $\quad \forall x \in \mathbb{R} \ni x>0,$

$\forall x>0 \ \exists y \ni y^2=x.$

Every real # has a cube root.

$\forall x \ \exists$ cube root ?! $\quad \forall x \ \exists y \ni y^3=x.$

For every # there is a larger #.

$\forall x \ \exists y \ni y>x.$

There is a largest real number.

$\exists x \ni \forall y, \ x>y.$

Two more:

If $x>1$, then $x^2>1$.

$\forall x>1$, $x^2>1$.

$(\forall x \ni x>1)$

There is no square root of -2 in $\mathbb{R}$.

$\not\exists\, x \ni x^2=-2$. (slang...)

$\forall x\in\mathbb{R}$, $x^2\neq -2$.

Negation of statements with quantifiers is tricky...

In words: the negation of "every day is sunny" isn't "every day is raining"!

It's "At least one day is not sunny." or "At least one day is raining."

[Here $\sim$sunny = raining]

Symbollically

- Negation of $\forall x,\ p(x)$

  is $\exists x,\ \sim p(x)$.

i.e. $$\boxed{\sim[\forall x,\ p(x)] \Leftrightarrow \exists x \ni \sim p(x)}$$

- Also $$\boxed{\sim[\exists x \ni p(x)] \Leftrightarrow \forall x,\ \sim p(x)}$$

__Ex__ Negate these stmts:

(a) $\forall x,\ g(x)<0$

$\exists x \ni g(x) \geq 0.$

(b) $\exists x \ni f'(x)=0$

$\forall x,\ f'(x) \neq 0.$

(c) $\forall \varepsilon>0\ \exists \delta>0 \ni 0<|x-a|<\delta \Rightarrow |f(x)-f(a)|<\varepsilon.$

$\exists \varepsilon>0 \ni \forall \delta>0\ \exists x \ni 0<|x-a|<\delta \wedge |f(x)-f(a)| \geq$