§14 Compact Sets (A quick tour)

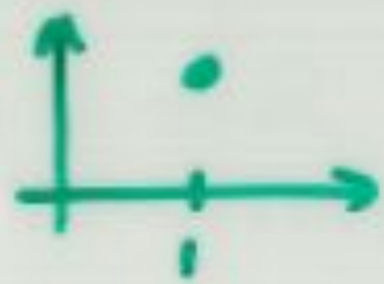
If you're working with fns $f: S \subseteq \mathbb{R} \to \mathbb{R}$ and could pick any kind of domain what would you choose?

① open int? have asymptotes

$$f: (0,1) \to \mathbb{R}$$
$$x \mapsto \frac{1}{x}$$

② single pt?

$$f: \{1\} \to \mathbb{R}$$
$$f(1) = \pi$$



Not terribly useful....

In gen'l we like:

<u>Def</u> $S \subseteq \mathbb{R}$ is <u>compact</u> if it is <u>closed</u> and <u>bounded</u>

This is <u>NOT</u> def$^{\underline{n}}$ of compact, but rather a characterization of compact given by HEINE-BOREL THEOREM.

<u>Why are they nice?</u>

- compact (cp) sets in $\mathbb{R}$ are bdd, so they have sup's, inf's.
- Because cp sets are <u>closed</u>, the sup and inf turn out to be <u>in</u> the set (hence max, min).
- Don't stretch off to +/- ∞

Example: In Calculus, "Closed Int Meth

C.I.M. A continuous fn always attains an absolute min, max on a closed interval $[a,b]$. Find it by:

- check critical pts ($f'=0$) in (a,b).
- check endpts.

Restated: Let $K \subseteq \mathbb{R}$ be compact, $f: K \to \mathbb{R}$ continuous. Then $f(K)$ attains its min and max (i.e. glb/inf, lub/sup).

$$C'(r) = 7x^2 - 4x = 0.$$
$$x(7x-4) = 0$$
$$x = 0 \text{ or } x = 4/7.$$

Bolzano - Weierstrass Thm (14.6)

If bounded $S \subseteq \mathbb{R}$ contains ∞'ly many points, $\exists$ at least one accumulation pt of S.