Exam 2 covers Sections 2.3–2.4 and Sections 3.1–3.4, with the following caveats:

- In Section 3.4, you are not responsible for "accumulation points" or "closures."
- You are not required to memorize the 15 different axioms in Section 3.2. You can focus on the writing exercises assigned for this section to know what will be important on the exam.

The test itself will have a mixture of short answer questions, calculations and proofs. To study for this test you should learn the definitions and theorems, but **there is no substitute for doing problems**. You might understand the definitions of injective and surjective, but if you haven't done a bunch of proofs that certain functions have those properties, you will be hard pressed to do so on the test. Similarly, if you haven't done problems which require you to come up with your own bijections to demonstrate two sets are equinumerous, or determined the interior and boundary of various subsets of $\mathbb{R}$, it will be tough to figured them out on the exam.

As I said in class, this exam will be more difficult because of the nature of the material, but we're not aiming to surprise you on the midterms; if a topic shows up on the exam, it's because we think it's important – and if we think it's important, it's something that should have come up in lecture, in discussion, in skills problems, perhaps on a writing quiz, and so on. I glanced back at a previous Exam 2 in this class, and here's what I determined about the "source" of each problem:

(1) This was a collection of definitions and examples used in lecture, homework and quizzes.
(2) This was a writing exercise and a writing quiz problem.
(3) This was also a writing exercise and quiz problem.
(4) A variant of a skills problem and examples from lecture, plus statements of definitions (as on some quizzes)
(5) A variant of examples and skills problems.
(6) This was a variant of an example done in lecture (which itself was presented as an example from a previous midterm).

As you can see, the skills problems, writing problems, and lectures are a good indication of what we consider important. Look through the comments written on your quizzes and make sure you understand why you lost points. Redo those problems, and then compare them to the posted solutions online. Compare your skills problems solutions to the online solutions as well. Then do similar problems from the book. Keep three things in mind:

It's easy to practice what you already know. It's a lot more fun to practice the types of problems that you're good at. But after making sure you're still ok with those concepts, you'll get much more benefit from studying those concepts and problems which you find harder. If you work on those this weekend and early next week, you can come to office hours with specific questions about where you got stuck. We can give you much better help if you show us what you've written down so far and where you're stuck, instead of a general "I don't understand this section." If nothing else, write down the definitions for the concepts in the problem, and see if that jogs anything in your memory.

Don't just look at the online solutions. It's not enough to read through our solutions to see if they make sense. You need to be able to come up with solutions *on your own.*

Be able to do problems without other resources. If you're stuck on a certain type of problem or concept, you should absolutely use lecture notes, examples in the textbook, and online solutions to help you figure it out. But you won't have those resources available on the exam, so you want to make sure you can eventually do similar problems with your book/notes closed, phone/laptop/tablet off, etc.

Review Problems

Here are some review problems for Section 3.4: [Update: Problem (2) has been fixed.] (11/6/16)

Book: The True/False statements in 1 and 2 are a good check. You can ignore 2(c,d,e,f). Problems 3 and 4 give a chance to find interiors and boundaries, and in Problem 5 you have to conclude whether a set is open or closed.

(1) Using our definition of open set ($S \subseteq \mathbb{R}$ is open if every $s \in S$ is an interior point of S), prove the following:
   (a) The union of two open sets is open. Can you use the same method of proof to show *any* union of open sets is open? Why or why not?
   (b) The intersection of two open sets is open. Can you use the same method of proof to show *any* intersection of open sets is open? Why or why not?

(2) Using our definition of closed set ($S \subseteq \mathbb{R}$ is closed if $S^C = \mathbb{R} \setminus S$ is open), prove the following:
   (a) The intersection of two closed sets is closed. Can you use the same method of proof to show *any* intersection of closed sets is closed? Why or why not?
   (b) The union of two closed sets is closed. Can you use the same method of proof to show *any* union of closed sets is closed? Why or why not?

As far as the other sections, if you follow the advice above, you'll have plenty of problems to work on. If you'd like some more, I've posted some extra review problems written by the TAs in a previous semester. Also, you might recall that the solutions to the exams from the Spring 2009 semester are online. That course used a different textbook and therefore covered material in a slightly different order, but the following questions from the first exam are relevant when studying Sections 3.1 and 3.3 for our midterm.

Spring 2009 Exam 1. (Solutions at `http://www.math.umn.edu/~keynes/3283Exam1Solutions.pdf`.)

4. Use induction to prove $5^n + 6^n < 7^n$ for all $n > N$, for some N. (Hint: this fails for $n = 1$, for example, so you must determine the lowest value for which the statement holds, and use that as the basis for your induction.)

5(a). Let $A = \left\{ \frac{n-1}{n+1} \,\middle|\, n \in \mathbb{N} \right\}$.
   i. Show that A is bounded. (i.e. bounded above *and* below)
   ii. Find $L = \inf A$ and $M = \sup A$ (no proof needed for this part). Determine whether or not $L \in A$ and $M \in A$.
   iii. Prove the value $M = \sup A$ you found above is in fact the least upper bound of A.

5(b). Determine if the following sets are bounded above or below. In each case, if the set is bounded above, find the supremum. If the set is bounded below, find the infimum.
   i. $\left\{3 + \frac{1}{2}, -2 + \frac{1}{2}, 3 + \frac{1}{4}, -2 + \frac{1}{4}, 3 + \frac{1}{8}, -2 + \frac{1}{8}\right\}$.
   ii. $\left\{x \in \mathbb{R} \,|\, x > 0 \text{ and } x^2 - 4x + 3 > 0\right\}$
   iii. $\left\{x \in \mathbb{R} \,|\, x^3 - x < 0\right\}$
   iv. $\{1 - .3, 2 - .33, 3 - .333, 4 - .3333, 5 - .33333\} \cup \left\{ \frac{1}{\sqrt{n}} \,\middle|\, n \in \mathbb{N} \right\}$