§1.1 Logical Connectives

Math consists of statements, sentences which can be classified as true or false — although we might not know which!

Ex Which of these are statements?

$p: 2+2=4$ Yes (T)

$q: 3+3=10$ Yes (F)

r: We're in Wisconsin Yes (F)

s: This is your favorite class
"Yes" assuming we can rank classes

t: $x^2-4x+3=0$ Yes – truth value depends on x.
(underlying assumption is that x is a #)

u: This sentence is false. Not a stmt

v: It's cold outside. "Yes" – depending on def$^{\underline{n}}$s.

w: Truth is beauty. Probably not...

Last time: A statement is a sentence (perhaps comprised of math'l expressions) which is either true or false — and it doesn't matter if we have all the information needed to know which it is.

Ex The temp. here is exactly 72°F

(Even if we don't know exact temp)

assumption that x is a # → $2x-7=3$

(Truth val. depends on value of x)

NOT: This sentence is false.

Can't be true, can't be false

can't have a truth value

Given stmts p, q we can create new stmts using logic operators. (term: sentential operators)

① Negation ($\neg$, $\sim$)

$\neg p$ ("not p") is true when p is false, false when p is true.

We can represent this with a "truth table."

p	$\sim p$
T	F
F	T

② Conjunction (and, $\wedge$)

$p \wedge q$ true only when p, q both true

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

③ Disjunction (or, $\vee$)

$p \vee q$ if p is T, q is T <u>or</u> both

(at least one is T)

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

There is also XOR ($\veebar$): exactly one is true

4 Implications.

(if p, then q, $p \Rightarrow q$)

p = antecedent (hypothesis)
q = consequent (conclusion)

p	q	$p \Rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Convention: $p \Rightarrow q$ false <u>only</u> when p is T, q is F.

<u>Ex</u> Determine truth values.

T If $\underbrace{\text{2 is positive}}_{T}$, then $\underbrace{\text{4 is even}}_{T}$.

F If $\underbrace{\text{3 is odd}}_{T}$, (then) $\underbrace{\text{pigs can fly}}_{F}$

T If $\underbrace{\text{pigs can fly}}_{F}$, then $\underbrace{\text{I'm a rock star}}_{T/F}$.

We can combine these operations

p: Jim is tall

q: Jim has red hair

$p \wedge q$ = p and q = (Jim is tall) and (Jim has red hair)

= Jim is tall and has red hair.

$\sim(p \wedge q)$ = not (Jim is tall and has red hair)

not $\sim p \wedge q$, which would be $(\sim p) \wedge q$

= (Jim is not tall) or (Jim does not have red hair)

added later, after the next slide

You try: Construct truth table for $\neg(p \vee q)$

p	q	$p \vee q$	$\neg(p \vee q)$	$(\neg p)$	$(\neg q)$	$(\neg p) \wedge (\neg q)$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

same truth values - "logically equivalent"

We've shown one of the two DeMorgan's Laws:

$\neg(p \vee q)$ is logically equivalent to $(\neg p) \wedge (\neg q)$

$\neg(p \wedge q)$ is logically equivalent to $(\neg p) \vee (\neg q)$

Ex Write the negations of these stmts:

* (Either) $x=0$ or $y<5$.

(not $x=0$) and (not $y<5$)

$x \neq 0$ and $y \geq 5$

n is prime and (n is) odd

n is not prime or n is even

n is composite or even

* For some, "either A or B" means "exclusive or," i.e "A or B but not both." But this isn't universal. Because XOR is rare for us, it's safest in 3283W to assume "inclusive or" unless told otherwise.

More combinations...

Def the contrapositive of $p \Rightarrow q$ is $(\sim q) \Rightarrow (\sim p)$

The converse of $p \Rightarrow q$ is $q \Rightarrow p$

Ex If $x > 1$, then $x^2 > 1$ T

CP: if $x^2 \leq 1$, then $x \leq 1$ T
C: if $x^2 > 1$, then $x > 1$ F

If it's raining, the sidewalk is wet. T

(Assume $\neg$wet is dry, $\neg$raining is sunny) CP: dry sidewalk $\Rightarrow$ sunny T
C: wet sidewalk $\Rightarrow$ raining T

KEY: CP logically equiv. to original implication.
(Converse isn't.)

Finally, a word about biconditionals...

A biconditional can be true or false

(We'll talk about this on Monday 9/12
and then continue to Section 1.2)