

The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

1.2 #4 The English language allows us to write the same phrase in many different ways. Your answers may be phrased differently but still be correct; ask us if you're not sure.

- (a) At least one person does not like Robert.
- (b) No student works part-time.
- (c) There exists a square matrix which is triangular.
- (d) For all $x \in B$, $f(x) \leq k$.
- (e) $x > 5$ and $3 \leq f(x) \leq 7$
- (f) x is in A , and for all y in B , $f(x) \geq f(y)$.

1.2 #5 Answers are in the back; talk to us about any questions or if you're not sure why certain statements are true or false.

1.2 #11 Again, answers are in the back. Talk to us if you're having difficulty parsing these nested quantifiers, because it's much easier to help you in person than in written solutions. Two hints: (1) think of "there exists" as "can I choose ... to make the following work?" (2) Parse the quantifiers in order, from left to right. So if you have " $\forall y \exists x \dots$ " your choice of x can depend on y , but in " $\exists x$ such that $\forall y$," you don't know what y is before choosing x . So, for example, in reverse order:

- (c) In words: "For all x and y , can I choose z such that $y - z = x$?" This is TRUE, because I already know what x and y are, and would choose $z = y - x$ to make the equation work.
- (b) In words: "Can I choose x and y such that, for all possible z , $x + y = z$?" This is FALSE, because I don't know the value of z before choosing x and y , and although one value of z would make it work, the equation would fail for all other values of z .

1.3 #1 (c) False; the inverse of $p \Rightarrow q$ is $\sim p \Rightarrow \sim q$. We don't use the inverse of an implication very often, because it's not logically equivalent to the original implication.

- (d) False; you have to show the statement holds for all possible values of n .
- (e) True.

1.3 #2 (c) True! The negation of "something is always true" is "it's false at least once."

- (d) False! To prove the statement is false, you would have to show that $p(n)$ is never true, no matter the value of n .

1.3 #3 (a) If no violets are blue, then at least one rose is not red.

- (b) If A is invertible, then there are no nontrivial solutions to $A\mathbf{x} = \mathbf{0}$.

- (c) If $f(C)$ is not connected, then f is not continuous or C is not connected.