Homework #4 Math 3283W - Fall 2016

The following problems will be relevant for your writing quiz on Thursday, 10/13/16.

Skill / Computational Problems. These problems are not handed in or graded, and do not involve enough writing to be the basis of a writing quiz, but are a good way to check that you understand the concepts used in the writing problems.

This week I've written the problems out on in this document, but some can be found in the book.

(1) Suppose A has exactly two elements and B has exactly three. How many functions are there from A to B? How many are surjective? How many are injective?

(2) How does the previous question change if the functions are now from B to A?

(3) Classify each function as injective, surjective, bijective or none of these.

  (a) $f : \mathbb{N} \to \mathbb{N}$ defined by $f(n) = n + 3$.

  (b) $f : \mathbb{Z} \to \mathbb{Z}$ defined by $f(n) = n - 5$.

  (c) $f : \mathbb{R} \to \mathbb{R}$ defined by $f(x) = x^3 - x$.

  (d) $f : \mathbb{N} \to \mathbb{Z}$ defined by $f(n) = n^2 - n$.

  (e) $f : [3, \infty) \to [5, \infty)$ defined by $f(x) = (x - 3)^2 + 5$.

  (f) $f : \mathbb{N} \to \mathbb{Q}$ defined by $f(n) = 1/n$.

(4) In each part, find a function $f : \mathbb{N} \to \mathbb{N}$ that has the desired properties.

  (a) Surjective, but not injective

  (b) Injective, but not surjective

  (c) Neither surjective nor injective

  (d) Bijective.

(5) Find counterexamples to each of these statements for $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$:

  (a) If f and g are surjective, then $f + g$ is surjective.

  (b) If f and g are surjective, then $f \cdot g$ is surjective.

Writing Problems. Your writing quiz on Thursday will be based on the problems below. A problem on the quiz could appear exactly as stated in the book, or it could be a slightly modified version of a problem below.

(6) Recall our notation for function composition: if the range of f is contained in the domain of g, then $g \circ f$ is the function defined as $g \circ f(x) = g(f(x))$. Prove the following statements:

  (a) If $f : A \to B$ and $g : B \to C$ are both surjective, then $g \circ f$ is surjective.

  (b) If $f : A \to B$ and $g : B \to C$ are both injective, then $g \circ f$ is injective.

(7) Now suppose $f : S \to S$ for some set S. Prove: if $f \circ f$ is injective, then f is injective.

Notice that exercises with a star have answers or hints in the back of the book. If those problems are assigned, use the back of the book to check your work. If a similar problem is assigned, you can do the starred problem to check whether you understand the concepts.