The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

3.1 #9: For the base step, $n = 1$, we have $1 = 2^1 - 1$, which is true. Now assume the formula is true for some $k \in \mathbb{N}$ and prove it for $k+1$:

$$\begin{aligned} 1 + 2 + 2^1 + \cdots + \cdots 2^{k-1} + 2^k &= \left(1 + 2 + 2^1 + \cdots + \cdots 2^{k-1}\right) + 2^k \\ &= \left(2^k - 1\right) + 2^k \\ &= 2 \cdot 2^k - 1 \\ &= 2^{k+1} - 1 \end{aligned}$$

as desired.

3.1 #14: For the base case, $n = 1$, we have $9^1 - 4^1 = 5$, which is certainly a multiple of 5. Now assume that $9^k - 4^k$ is divisible by 5 for some $k \in \mathbb{N}$, and use that knowledge to prove $9^{k+1} - 4^{k+1}$ is divisible by 5 as well. We do that by rearranging, factoring, and adding 0 in a clever way, namely $0 = -9 \cdot 4^k + 9 \cdot 4^k$:

$$\begin{aligned} 9^{k+1} - 4^{k+1} &= 9^{k+1} - 9 \cdot 4^k + 9 \cdot 4^k - 4^{k+1} \\ &= 9(9^k - 4^k) + 9 \cdot 4^k - 4 \cdot 4^k \\ &= 9(5m) + (9-4) \cdot 4^k \\ &= 5(9m) + 5 \cdot 4^k \\ &= 5(9m + 4^k) \end{aligned}$$

Notice where we made use of the assumption that $9^k - 4^k$ is divisible by 5, by replacing $9^k - 4^k$ with $5m$ for some integer m. But then we've shown that $9^{k+1} - 4^{k+1} = 5(9m + 4^k)$, which is divisible by 5.

Note that this is almost exactly Example 3.1.4 with 7, 4, and 3 replaced by 9, 4, and 5.

3.1 #17: (a) The simplest way to see how the induction step fails is to imagine carrying this proof out with two marbles of different colors. When you take away one marble, it's true that "all" of the remaining marbles (i.e. just one of them) are the same color. But that doesn't mean the two marbles you started with have to be the same color.

(b) In this proof the base step fails. When $n = 1$, you can check that $n^2 + 7n + 3 = 11$, which is odd.

3.3 #3,4:

(e) In order, the sup, max, inf and min are: $1/2$, $1/2$, 0, does not exist.

(g) In order, the sup, max, inf and min are: 1, does not exist, $1/2$, $1/2$.

(l) In order, the sup, max, inf and min are: 2, does not exist, 0, does not exist.

(m) In order, the sup, max, inf and min are: 5, 5, does not exist, does not exist.

(n) In order, the sup, max, inf and min are: $\sqrt{5}$, does not exist, does not exist, does not exist.