The following is a *non-comprehensive* list of solutions to the computational problems on the homework. For some problems there is a sketch of a solution. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work. Feel free to talk to us about any of the writing intensive problems.

Please let us know if you spot any typos and we'll update things as soon as possible.

4.1 #11 Here's one direction of the proof. Suppose s_n converges to s, and let's prove that t_n converges to s as well. Let $\varepsilon > 0$. Because $s_n \to s$, there exists N such that $n > N$ implies $|s_n - s| < \varepsilon$. Then $|t_n - s| < \varepsilon$, since $t_n = s_{n+k}$, and $n + k > N$.

4.2 #3 The limit in (a) is 5/7. The limit in (b) is 0. Ask us if you're not sure how to justify those values.

4.2 #6 (a) This is false. $s_n = n$ and $t_n = -n$ diverge to $+\infty$ and $-\infty$, respectively, but $(s_n + t_n) = (0, 0, 0, \ldots)$ converges to 0.

(b) This is false. Let $(s_n) = (1, 0, 1, 0, 1, 0, \ldots)$ and $(t_n) = (0, 1, 0, 1, 0, 1, \ldots)$. These sequences both diverge, but $(s_n t_n) = (0, 0, 0, \ldots)$ converges to 0.

(c) This is true, using the limit laws:

$$\lim t_n = \lim(s_n + t_n - s_n) = \lim(s_n + t_n) - \lim s_n.$$

(d) This is false. Let $s_n = 0$ and $(t_n) = (-1, 1, -1, 1, \ldots)$.

4.2 #8 (a) $s_n = 1/n$ converges to 0, and $\dfrac{s_{n+1}}{s_n} = \dfrac{n}{n+1} \to 1$. There are lots of other examples.

(b) $t_n = n$ diverges to $+\infty$, and $\dfrac{t_{n+1}}{t_n} = \dfrac{n+1}{n} \to 1$. There are many other examples.

4.3 #4 There are lots of examples of each. Here are some possibilities. You can ask us if you're having trouble justifying any of these answers.

(a) $s_n = (-1)^n/n$. (We need something which converges but isn't increasing or decreasing.)

(b) $s_n = n$. (We need something which increased but doesn't converge.)

(c) $s_n = (-1)^n$. (We need something which is bounded but doesn't converge.)

4.3 #5 These are false. Remember that one can be increasing, and the other decreasing!

(a) With a little experimenting, you can find a counterexample. The easiest way for me is a little artificial – figure out a few numbers at the beginning of each sequence which make things brake. For example, if

$$(a_n) = (0, 1, 4, 4, 4, 4, \ldots)$$
$$(b_n) = (3, 1, 0, 0, 0, 0, \ldots)$$

Then $(a_n + b_n) = (3, 2, 4, 4, 4, 4 \ldots)$ which isn't monotone.

(b) With a little experimenting, you can find a counterexample. For example, if $a_n = (3/2)^n$ (increasing) and $b_n = 1/n$ decreasing, you can check that $a_1 b_1 \geq a_2 b_2$ but $a_3 b_3 \leq a_4 b_4$.

4.3 #8 This one is a little tricky. Start by figuring out when $s_1 \leq s_2$, i.e. $k \leq \sqrt{4k-1}$. If you work out the algebra, you get

$$k \in (2 - \sqrt{3}, 2 + \sqrt{3})$$

So for those values of k, you you know $s_1 \leq s_2$, which is the base case for an inductive proof that s_n is increasing. Can you finish the proof and show that s_n is increasing in this case?

You can do something similar for decreasing, and find $k \geq 2+\sqrt{3}$. (The algebra also suggests $1/4 < k < 2-\sqrt{3}$ works, but you have to be a little careful. Try starting out with $s_1 = 1/4$, and you'll find that very quickly, the numbers inside the square root become negative.)