Math 3283W Fall 2016 - Writing Quiz #7
Thursday 11/3/16 - 20 minutes **Name:** Key

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

Definitions. This portion of your quiz will be graded for mathematical correctness only.

(1) (3 Points) Complete the definition: Given a set $S \subset \mathbb{R}$, $m = \min S$ if...

$m \in S$ and $\forall s \in S,\ m \leq s$.
(+1) (+2)

(2) (2 Points) Complete the definition: Given a set $S \subset \mathbb{R}$, M is an *upper bound* of S if...

$\forall s \in S,\ s \leq M$.

(3) (2 Points) Complete the definition: Given a set $S \subset \mathbb{R}$, $M = \sup S$ if...

M is an upper bound and no smaller number is an upper bd
Symbolically: if $m<M$, $\exists s \in S \ni m<s$
} +1 (either version is ok)

Writing. This portion of your quiz will be graded for both writing and correctness.

(4) (8 Points) Suppose that $x_1, x_2, \ldots, x_n$ are real numbers. Prove $|x_1 + x_2 + \cdots + x_n| \leq |x_1| + |x_2| + \cdots + |x_n|$.

We'll use induction. (Note: for this proof it's ok to assume $\triangle$ ineq for two #'s)
+1 for method

+2 for base case (n=1 or n=2 ok mathematically, although clarity of what's being assumed could affect writing score.)
When $n=1$, $|x_1| = |x_1|$.
When $n=2$, $|x_1+x_2| \leq |x_1| + |x_2|$ by triangle inequality.

Now assume $|x_1+\cdots+x_n| \leq |x_1|+|x_2|+\cdots+|x_n|$. Then +2 for assumption

$$|x_1+x_2+\cdots+x_n+x_{n+1}| = |(x_1+\cdots+x_n)+x_{n+1}|$$
+1 using $\triangle$ ineq (e.g. n=2)
$$\leq |x_1+\cdots+x_n| + |x_{n+1}|$$
+2 using induction assumption
$$\leq |x_1|+|x_2|+\cdots+|x_n|+|x_{n+1}|$$

as desired.

Other orders/arrangements/methods possible; scored similarly (e.g. 5 pts total for everything after base case)