Math 3283W Fall 2016 - Writing Quiz #9
Thursday 12/1/16 - 20 minutes Name: Solutions

Remember: your work on in the "writing" portion of this quiz will be graded on the quality of your writing and explanation as well as the validity of the mathematics. (5 Points)

Definitions. This portion of your quiz will be graded for mathematical correctness only.

(1) (3 Points) Complete the definition: s_n is *increasing* if...

for all n, $s_n \le s_{n+1}$

(2) (3 Points) Complete the definition: s_n is *monotone* if...

s_n is increasing or decreasing

Writing. This portion of your quiz will be graded for both writing and correctness. Use the back of this sheet to complete your solution if necessary.

(3) (9 Points) Prove the following sequence is monotone and bounded, and find its limit:

$$s_1 = 2 \text{ and } s_{n+1} = \sqrt{2s_n + 3}. \qquad \text{(Hint: } \sqrt{7} < 3.\text{)}$$

First, we'll use the Monotone Convergence Theorem (MCT) to show s_n converges. We need to show s_n is bounded and monotone.

+3 pf that s_n incr'g

+1 bdd below

+3 pf that s_n bdd above

+1 using MCT

+1 find s.

We can use induction to prove s_n is increasing. For the base case we note that $s_1 = 2 \le \sqrt{7} = s_2$. Next assume $s_{k+1} \ge s_k$. Then

$$s_{k+2} = \sqrt{2s_{k+1}+3} \ge \sqrt{2s_k+3} = s_{k+1}$$

Thus s_n is increasing.

Next we show s_n is bounded. Because it's increasing, s_n is bounded below by $s_1 = 2$. We can prove s_n is bounded above by 3 using induction. First, $s_1 = 2 < 3$, which establishes the base case. Next, if $s_k \le 3$, then

$$s_{k+1} = \sqrt{2s_k + 3} \le \sqrt{2\cdot 3 + 3} = \sqrt{9} = 3.$$

(other grading schemes possible)

Thus s_n is bounded and increasing, so by the MCT $s_n \to s$ for some s. Because $\lim s_n = \lim s_{n+1}$, we have

$$s = \sqrt{2s+3}$$
$$s^2 = 2s+3$$
$$s^2 - 2s - 3 = 0 \quad \Rightarrow \quad s = 3, -1$$

Because $s_1 = 2$ and s_n is increasing, we conclude $s = 3$.