

Exam 2 covers the material from the first midterm until this week – Chapters 7, 8, 9 and 10, with a portion of Chapter 11, as described below.

As far as the type of problem, you can expect a mixture similar to the first exam. Most will be very similar to homework problems and/or propositions or theorems which we devoted a lot of class time to. Somewhere on the exam you'll be asked to prove something we haven't done in class – say, given some information about a triangle or quadrilateral, prove that two given points are equal, etc. *Don't freak out when you read this problem and think "I've never seen this!"* That's the point: to check if you understand the definitions and techniques well enough to apply them in a slightly different setting. It won't be worth a huge percentage of the score.

In general, I'm much more interested in whether you can explain concepts than regurgitate facts. I couldn't care less if you've memorized a long formula for the circumcenter of  $\triangle ABC$ . But I'm very interested in whether you can use what you know about perpendicular bisectors to explain how you know the perpendicular bisectors of the sides of  $\triangle ABC$  must meet in a point.

As before, the exam is closed book, closed notes, closed calculator, etc. Don't trivialize a problem, meaning if I ask you to prove a result from class, you shouldn't just cite the theorem.

A few words of advice, by chapter:

**Chapter 7:** In addition to the basic definitions in the chapter, you should know what medians, altitudes, perpendicular bisectors and angle bisectors are, and where they meet. There are various "concurrency" theorems and formulas. Proving the perpendicular bisectors are concurrent (and explaining why the circumcenter is the center of the circumcircle) is pretty reasonable for an exam. Same for the angle bisectors and incenter. Proving the medians are concurrent is a cute little application of parametric lines, but I'm not so interested in whether you can memorize a bunch of expressions with  $1/2$ 's,  $1/3$ 's and  $2/3$ 's. I'd probably be more interested in applications of the median, like the homework problem that showed the medians split a triangle into six smaller (not necessarily congruent) triangles which all have area  $1/6$  that of the original triangle.

Under no circumstances should you start memorizing the formulas for barycentric coordinates of the feet of the altitudes, the orthocenter, etc. Don't memorize formulas for the circumradius, or inradius. We can be glad somebody has figured those out, but I have other things I can test you on.

Suggested review problems: any of the HW, plus 7.34

**Chapter 8:** Major results like Proposition 8.4 are important; results like Theorem 8.5 or Propositions 8.6 and 8.7 are useful, but rather than memorizing them you could aim to become familiar enough with the methods of this chapter so you could prove them if needed. One suggestion I can make is that you shouldn't be afraid of "vector methods" – i.e. doing calculations involving the vertices of a quadrangle or the vectors between them. [See the proofs we did in class involving parts (iv) and (vi) of Proposition 8.4, for example. Or prove Theorem 8.4.] The parts of the chapter that we skimmed over in class (e.g. much of the material on circles) won't be emphasized on the exam. That doesn't mean circles won't appear, or that you can't use basic facts about circles (radius perpendicular to a tangent line, for example) but you wouldn't ever have to

prove Theorem 8.18, for example.

Suggested review problems: any of the HW, plus 8.21, 8.23, 8.25, 8.26, 8.29, 8.44

**Chapter 9:** You should know the definitions of conformal affinity and similarity, and be able to do the types of problems involving similar triangles that we did in class. The big (and new) topic in this chapter is circle inversion, which we've also referred to as "reflecting" across a circle. You should be rock-solid with the formula to invert  $X$  across a circle of radius  $\rho$  centered at the origin, as well as the geometric definition –  $X' = \mathcal{I}(X)$  is the point on  $\overrightarrow{OX}$  such that  $|\overline{OX}| \cdot |\overline{OX'}| = \rho^2$ . You should be able to use both of those definitions to find the reflection of any line or circle in the plane. Finally, you should be able to do all of that (definitions, images of lines and circles) if the mirror is centered somewhere other than the origin, as well. You should be aware that circle inversion is conformal, and what that means, but you needn't be able to prove it.

Suggested review problems: any of the HW, plus 9.6, 9.11, 9.13, and 9.6 with mirror  $(x + 1)^2 + y^2 = 4$ .

**Chapter 10:** Do **not** spend time memorizing the axioms in this chapter. If I want you to say anything an axiom, I'll provide it for you on the exam. The basic definitions and properties of lines and distances in the Poincaré Half Plane comprise the important material in this chapter. Because you won't get the relevant homework back before the exam, any questions from Chapter 10 should closely resemble things we did in class. You don't need to memorize the formulas 10.6 and 10.7 on page 171. If I want you to use those, I'll provide them.

Suggested review problems: the posted homework

**Chapter 11:** We covered the distance formula in Theorem 11.1 back in Chapter 10, but you don't need to memorize it. You should be familiar with Poincaré Direction indicators, but don't need to know the cross ratio formulas to determine whether two Poincaré lines intersect, or any of the material we covered about angles.

Suggested review problems: the posted homework

I've included a lot of possible review problems. The goal is not for you to do every single one of them and memorize their solutions. Pick and choose as appropriate; if you're very confident with chapter 8 you don't need to spend much time there. If you're not as sure of chapter 9 you might want to look at a few more of those review problems. As always, please feel free to email me with questions.