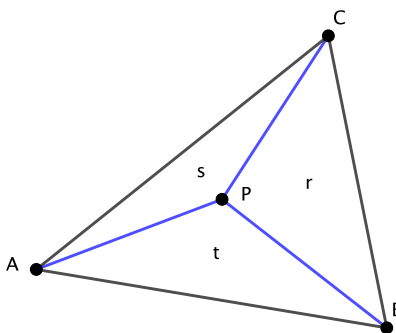


In the last week we learned about *barycentric coordinates*. Given any triangle $\triangle ABC$, it turns out you can write any point $X \in \mathbb{R}^2$ as a unique linear combination of the vertices:

$$X = rA + sB + tC,$$

where r , s and t satisfy the condition $r + s + t = 1$. These numbers are the *barycentric coordinates of X based on $\triangle ABC$* , and we'll sometimes write $X = (r, s, t)^{\triangle ABC}$ or $(r, s, t)^{\triangle}$ for short.

We used barycentric coordinates to prove an important theorem, and you learned what the relative values of r , s and t say about where the point $(r, s, t)^{\triangle}$ is located. This lab will let you experiment with another interpretation of barycentric coordinates **which is valid as long as r , s and t are nonnegative**. As illustrated in this diagram: if $P = rA + sB + tC$, where P is inside the triangle and $r + s + t = 1$, then r represents the area of triangle PBC as a percentage of the area of the whole triangle. Similarly, s represents the area of the triangle PAC as a percentage of the whole triangle, and t represents the percentage of PAB .



Be aware: the instructions below are written assuming that you're using a version of GeoGebra that you have downloaded to a computer, either **GeoGebra Classic**, or **GeoGebra Math Calculator**. There are slight differences between the two, but they won't matter for this lab. Many of you are using a tablet or online version of GeoGebra instead; that's fine, but be aware that the interface is slightly different. For example, when I right-click on a polygon, I get a pop-up menu which allows me to rename it. On the web version, the popup menu doesn't include this option. You have to choose **Settings** on the popup menu, which then gives you a chance to rename the object.

If you've been using that version for previous labs, you've probably already figured out these differences, but let me know – say, on Monday, not Wednesday at 12:05pm – if you have trouble following a step. Or you could download GeoGebra to a computer. In that case I'd recommend the **GeoGebra Classic** version, which provides a more standard computer application interface.

GEOGEBRA CONSTRUCTION

- (1) Open a new GeoGebra window. Turn off the grid and axes. Define a triangle with vertices $A = (-1, 1)$, $B = (2, -1)$ and $C = (3, 3)$. If this is the first thing you've done, GeoGebra will also show a dependent object called `poly1` which represents the area of this triangle. Right click on the name (or the interior of the triangle) and rename it `totalArea`.
- (2) Define a point P somewhere inside the triangle (e.g. $(1, 1)$).
- (3) Define the triangle `ta = Polygon[P, B, C]`; you can either enter this command in the Input Field below, or choose the Polygon tool and click on those points. (You can then rename the Polygon to `ta` in that case.)

Define triangles t_b and t_c as appropriate. You should now see the triangle ABC split into three smaller triangles which meet at P , similar to the picture above.

- (4) You are now ready to define the barycentric coordinates of your point P . In the input field below, enter

$$r = ta / totalArea$$

followed by appropriate definitions for s and t .

- (5) Now we're ready to display the barycentric coordinates. GeoGebra can display text labels that include the current values of variables (such as r , s and t). The process is a bit different depending on your version of GeoGebra, so for this step I'll include directions for each:

- **GeoGebra Classic (Downloaded):** Choose the **Text** tool, which is in the dropdown menu below the slider. In the textbox, enter *exactly* the following:

$$(r,s,t)^{\triangle} = (, ,)^{\triangle}$$

(If you prefer, you could use the **Symbol** drop down menu to insert the triangle symbol. The text `\triangle` is computer-ese – L^AT_EX code, actually – for “draw the triangle symbol.” When you enter the text, GeoGebra should activate the **LaTeX Formula** button automatically, but you can check that box if needed.)

Now you need to go fill in the empty coordinates in the second set of parentheses. Place the cursor in the first empty coordinate. Then click on the **Objects** button. This contains a list of all objects and variables that you've defined. Scroll down and choose r from the list. Then insert s and t into the second and third coordinates. Use the **Preview** window to check for any typos, and click **Ok** when you're finished.

- **GeoGebra Math Calculator (Downloaded):** The **Text** tool may be in the group of tools labeled with “Media.” Otherwise the process with this version of GeoGebra is nearly identical, except there is no **Objects** button; instead, click the **Advanced** arrow below; the list of objects is on the tiny tab with the GeoGebra icon.
 - **Online GeoGebra Classic or Graphing Calculator:** Essentially the same as the downloaded Math Calculator version.
- (6) At this point, clean up your diagram. Spend time right clicking on objects (or using the **Settings** or **Object Properties** menus) to hide any extraneous information or labels. For example, turn off the display of labels for segments and polygons. For full credit, your diagram should only include the labels A , B , C , r , s , and t , along with the text of the barycentric coordinates. You can also change the color of the triangles, turn their **Opacity** down to 0, so they're not shaded in, and so on. The point is to make a nice, uncluttered diagram to illustrate the ideas! Also, zoom in and/or adjust the size of the window so that the triangle and text label fill up the view; they shouldn't be crammed into a tiny corner of the window.
- (7) Test your construction by moving the points A , B , C , and P . What happens when you move P outside of the triangle? Or to one of the vertices, or line segments of the triangle? Is that consistent with what we learned on Wednesday about the “quadrants” of the barycentric plane? With that knowledge in mind, how do you know the displayed coordinates are not correct if P is outside of the triangle? (You don't need to send me answers to these questions.)

To receive credit for this assignment, save your file as `lastname-5335-lab2.ggb` and email it to me as an attachment by the beginning of class on Wednesday, 10/9/19. For full credit, make sure to follow the directions about above removing extraneous details. (Axes, grid, labels, etc.)