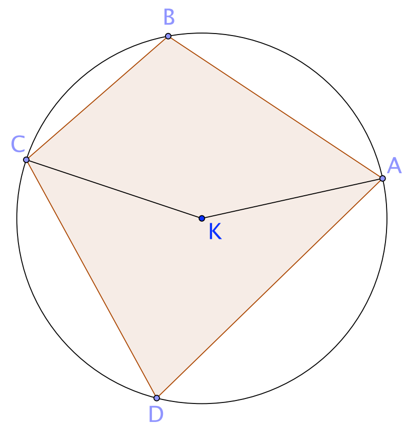
The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

6.5.10: Since $n = 6$, Theorem 14 tells us $s = 2r\sin(\pi/6) = 2r(1/2) = r$. (If you draw a picture of the regular hexagon this is clear, because the triangles you create by connecting the center to the vertices are equilateral, so of course $s = r$.) Hence the perimeter is $n \times s = 6r$.

6.5.17: This problem is an application of the first and third parts of Theorem 16. (The second case of Theorem 16, with a right angle, is really just a special case of the first one.) Here's our setup; I'm assuming K is on the same side of $\overline{AC}$ as D, or perhaps K is on $\overline{AC}$. By relabeling points I could always ensure that's the case.



By the first part of Theorem 16, $|\angle AKC| = 2|\angle ADC|$. By the third part, $|\angle AKC| = 2\pi - 2|\angle ABC|$. Thus $2|\angle ADC| = 2\pi - 2|\angle ABC|$; rearrange and divide by two to get $|\angle ABC| + |\angle ADC| = \pi$. Similar work shows that the other pair of vertex angles of $ABCD$ also add to π.

6.5.23: Let ABCD be *any* quadrangle. Then the midpoints are

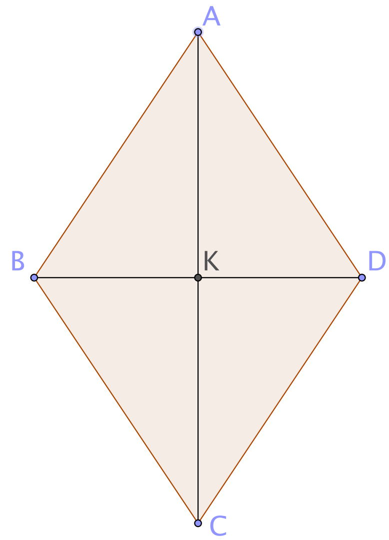
$$E = (A+B)/2$$
$$F = (B+C)/2$$
$$G = (C+D)/2$$
$$H = (A+D)/2$$

To show that $EFGH$ is a parallelogram we can show it satisfies any of the six conditions in Proposition 3. I think the fourth is easiest – that the diagonals of $EFGH$ bisect each other. We calculate the midpoints of the diagonals:

$$\frac{E+G}{2} = \frac{(A+B)/2+(C+D)/2}{2} = \frac{A+B+C+D}{4}$$
$$\frac{F+H}{2} = \frac{(B+C)/2+(A+D)/2}{2} = \frac{A+B+C+D}{4}$$

(Draw a picture of this!) Because the midpoints of the two diagonals are equal, the diagonals must bisect each other, so $EFGH$ is a parallelogram.

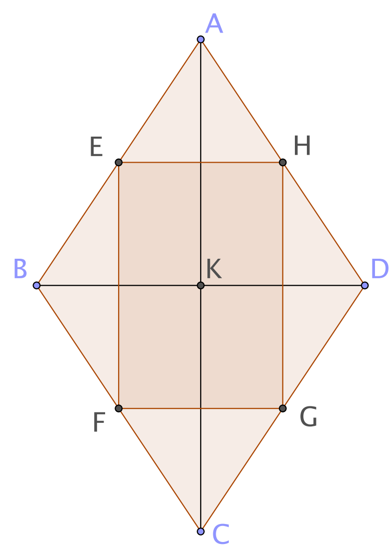
6.5.26: First let's prove that conditions (i) and (ii) are equivalent, using the following diagram.



Assume $ABCD$ is a rhombus. Because it's also a parallelogram (by definition), the diagonals bisect each other. Hence you can use the SSS congruence criterion to show that each of the four inside triangles are congruent to each other. In particular we must have four congruent angles surrounding the point K, meaning that each is a right angle. This proves (i) $\Rightarrow$ (ii).

Conversely, assume the diagonals are perpendicular. We still know $ABCD$ is a parallelogram whose diagonals must therefore bisect each other. So now we can show the four triangles are congruent using SAS and conclude the sides of $ABCD$ are all equal, proving (ii) $\Rightarrow$ (i).

Now add the midpoints of the sides to the picture:

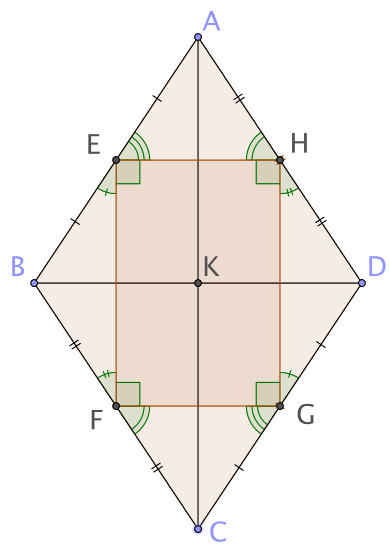


Suppose we know (i) or (ii) is true (and therefore both, by our work above!), and we want to prove (iii). If we can show any one of the vertex angles of $EFGH$ is a right angle then a symmetric argument will prove it for all of them, and $EFGH$ will be a rectangle. One way to show $|\angle EFG| = \pi/2$ is to show that $E - F \perp G - F$. Let's compute their dot product:

$$\langle E-F, G-F\rangle = \left\langle \frac{A+B}{2} - \frac{B+C}{2}, \frac{C+D}{2} - \frac{B+C}{2} \right\rangle = \left\langle \frac{A-C}{2}, \frac{D-B}{2} \right\rangle = \frac{1}{4}\langle A-C, D-B\rangle$$

But by (ii), $A - C \perp D - B$ so this dot product is zero.

To finish the proof we need to assume (iii) and show that (i) or (ii) (and hence both) are true. This is the trickiest part. Using the fact that E, F, G and H are midpoints together with the knowledge that $ABCD$ is a parallelogram and $EFGH$ is a rectangle (and hence has right angles at each vertex), I can label the following congruent sides and angles. (Ask me if you're not sure how to get these. This isn't an exhaustive picture, either; we still know that the diagonals of $ABCD$ bisect each other, for example!)



There are lots of ways to finish the problem now. Here's one. I claim that the diagonals of $ABCD$ bisect the sides of the rectangle $EFGH$. Consider the midpoint of E and H:

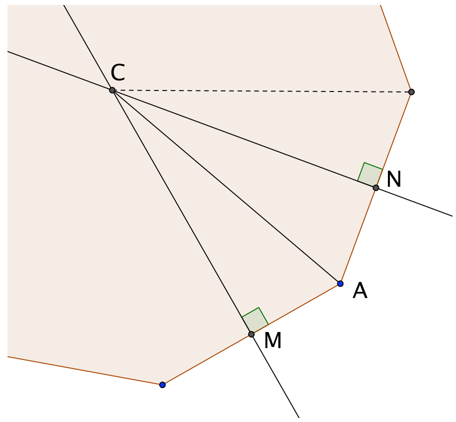
$$L = \frac{E+H}{2} = \frac{(A+B)/2 + (A+D)/2}{2} = \frac{2A+B+D}{4} = \frac{3A+C}{4}$$

where the last equality comes from the fact that $B + D = A + C$ (Proposition 3(vi)).

But that point, $\frac{3}{4}A + \frac{1}{4}C$ is on the line segment $\{A + t(C - A)\}$ from A to C! (Let $t = 1/4$.) Hence $\overline{LE} \cong \overline{LH}$. Furthermore, $\overline{AC}$ must bisect $\overline{FG}$ and because $EFGH$ is a rectangle, $\overline{AC}$ must in fact be perpendicular to $\overline{EH}$ and $\overline{FG}$. In other words, $|\angle ALE| = |\angle ALH| = \pi/2$.

Now we're just about done. By SAS, $\triangle ALE \cong \triangle ALH$. Hence the segments with one tick mark (such as $\overline{AE}$) must be congruent to those with two tick marks (such as $\overline{AH}$) and it quickly follows that $ABCD$ is a rhombus.

6.5.29: Let $\mathcal{P}$ be a regular polygon with an unknown number n sides. Following the hint in the back of the book, we start by drawing in the perpendicular bisectors of two adjacent sides. The bisectors intersect at a point in the interior of the polygon which I'll call C. Continuing to follow the hint, we connect C to the vertex between M and N:



Because M and N are the midpoints of the sides, $\overline{AM} \cong \overline{AN}$, and the two triangles we've formed share the side $\overline{AC}$. Hence by the SSA congruence criterion for right triangles (Theorem 10 in Chapter 5), $\triangle CMA \cong \triangle CNA$. In particular, $|\angle CAM| = |\angle CAN|$, from which we see that $\overline{AC}$ bisects $\angle MAN$. Also, $|\overline{CM}| = |\overline{CN}|$, as we're required to prove.

Now draw the segment from C to the next vertex, represented by the dotted line. By SAS (where the angle is the right angle at N), this new triangle is congruent to $\triangle CNA$. Now draw a dotted line representing the perpendicular bisector of the next side. (This dotted line isn't in my picture.) You can once again show this triangle is congruent to the others you've formed. At this point you can catch on to the pattern – you can go all the way around the polygon, forming $2n$ triangles which are all congruent, and whose sides are either (1) half of a side of the polygon, (2) the segment from the center point to the midpoint of a side, or (3) the segment from the center point to a vertex.

Together these facts are enough to prove the theorem. (Ask me if you want me to fill in the remaining details.)

6.5.49: Glancing over some of the homeworks it seems most people chose kites. Here's another possibility.