This is an open-book, -library, -internet take home exam. You are not allowed to collaborate; I am the only person you are allowed to consult. You can ask questions during office hours, or you can email me at any time during the day.

Because our previous midterms focused on Euclidean geometry, this final covers the geometry of the Poincare Half Plane.

As always, you should explain your work, writing complete sentences with reasonably correct grammar. A good rule of thumb is that the work you hand in for this exam should not be your first draft. Figure out the problem on another sheet of paper, organize your thoughts, and *then* write out your solution.

Due: Tuesday, 12/22/2009 at noon in my mailbox in Vincent 107.

(You may hand the exam in earlier if you wish.)

Problems

10.5.12: (14 Points)
10.5.28: (14 Points)
10.5.31: (16 Points)
10.5.38: (14 Points)

A: Prove: if $0 < \theta < 2\pi$, then there exists a hyperbolic quadrangle the sum of whose angles is θ. (14 Points)

B: In class we found the area of a hyperbolic regular pentagon whose interior angles were all $\pi/2$; this pentagon could therefore tessellate the Poincare Half Plane by arranging four of them around each vertex. It is a fact that, given an **even** integer $n > 2$, there exists a hyperbolic regular pentagon which can tessellate the Poincare Half Plane by arranging n of them around each vertex. For a given n, find the area of such a pentagon. (14 Points)

C: Let $\mathcal{D}_{0,s}(x, y) = (sx, sy)$ be the function which dilates the Poincare Half Plane by scaling radially away form the origin by a factor of s.

(i) Prove that $\mathcal{D}_{0,s}$ is an isometry of the Poincare Half Plane, i.e. prove that it preserves the distance between points. (7 Points)

(ii) Prove that $\mathcal{D}_{\omega,s}$ is an isometry of the Poincare Half Plane, where $\mathcal{D}_{\omega,s}$ performs the same scaling, but from $(\omega, 0)$ instead of $(0, 0)$. (7 Points)