The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

1.6.5: We discussed this representation of a line in class on 9/30, but you can prove it with just a bit of relabeling:

$$\begin{aligned}\overleftrightarrow{PQ} &= \{P + t(Q-P) \mid t \in \mathbb{R}\}\\ &= \{P + tQ - tP \mid t \in \mathbb{R}\}\\ &= \{(1-t)P + tQ \mid t \in \mathbb{R}\}\\ &= \{aP + bQ \mid a, b \in \mathbb{R} \text{ and } a+b=1\}\end{aligned}$$

Where the last line just comes from relabeling $(1-t)$ and t as a and b, respectively.

1.6.6: The given line goes through the point $(15, -8)$, and has a direction vector $(-7, -17)$. Hence the vector $(17, -7)$ is perpendicular to the line, and the normal form is:

$$\langle (17, -7), X - (15, -8)\rangle = 0$$

Other answers are possible; $(17, -7)$ may be replaced with any nonzero multiple of itself, you can distribute the dot product and have $\langle (17, -7), X\rangle = 311$, etc.

1.6.7: This problem is essentially the reverse of 1.5.6. From the given information, the line has a direction indicator U which is perpendicular to $(3, -14)$, so $U = (14, 3)$ would work. Finding a point on the line is a bit trickier. One possible approach is to guess that there might be a point on the line for which $X = (x_1, 0)$, in which case

$$\langle (4, -13), X\rangle = \langle (4, -13), (x_1, 0)\rangle = 4x_1 = 5$$

which means $x_1 = 5/4$ and imples that $X = (5/4, 0)$ is on the line. Hence a parametric form of the line would be $(5/4, 0) + s(14, 3)$.

1.6.9: Most people drew pictures for this problem, but there are ways to base your proof on definitions and propositions. One way is to use Proposition 16. So to prove that "being on the same side" of some given line $l : \{\langle A, X - Y\rangle = 0\}$ is an equivalence relation:

- $\langle A, P - Y\rangle$ and $\langle A, P - Y\rangle$ certainly have the same sign. (They're equal!)
- If $\langle A, P - Y\rangle$ has the same sign as $\langle A, Q - Y\rangle$ then $\langle A, Q - Y\rangle$ has the same sign as $\langle A, P - Y\rangle$.
- If $\langle A, P - Y\rangle$ has the same sign as $\langle A, Q - Y\rangle$ and $\langle A, Q - Y\rangle$ has the same sign as $\langle A, R - Y\rangle$, then $\langle A, P - Y\rangle$ has the same sign as $\langle A, R - Y\rangle$.

Conversely, "being on opposite sides" of l fails to be an equiavlence relation immediately, because a point P will not be related to itself: $\langle A, P - Y\rangle$ does not have a different sign than $\langle A, P - Y\rangle$. (Duh... but that needs to be written out!)

1.6.17: Draw a picture of four points P, Q, R and S and draw the line segment from Q to S to help understand this problem! You can prove this "quadrilateral inequality" by applying the triangle inequality twice.

$$|\overline{PS}| \leq |\overline{PQ}| + |\overline{QS}| \qquad \triangle \text{ inequality applied to } |\overline{PS}|$$
$$|\overline{PS}| \leq |\overline{PQ}| + |\overline{QR}| + |\overline{RS}| \qquad \triangle \text{ inequality applied to } |\overline{QS}|$$

The conditions to make *equality* hold will make sense if you draw a line segment on which P, Q, R and S appear in that order. If you're not sure how to prove those conditions, talk to me in office hours.

1.6.30: We defined lines to be parallel if they have direction vectors which are multiples of each other. So if $l = \{Q + sU\}$, then it's certainly true that the line $k = \{P + sU\}$ is parallel to l (since it has the same direction indicator) and is incident with P.

It remains to prove that this is the *unique* such line. So let

$$m = \{P + sV\}$$

be any line through P which is parallel to l. By definition of parallel lines, that means V (m's direction indicator) must be a multiple of U (l's direction indicator),

$$V = cU, \text{ some } c \in \mathbb{R}.$$

But U is also the direction indicator of k, meaning m has a direction indicator which is a multiple of k's. Hence $m||k$ and, since they share the point P, they must be the same line.

If you wish, you can use the third part of Proposition 3 to prove the lines are identical.

1.5.42: Here is one possible approach, which avoids any calculations with the components of U and V. First, assume $\langle U, V\rangle = 1$. To prove $U = V$ it suffices to show $||U - V|| = 0$ or, equivalently, $||U - V||^2 = 0$. Using Lemma 3,

$$||U - V||^2 = ||U||^2 + ||V||^2 - 2\langle U, V\rangle = 1 + 1 - 2(1) = 0$$

To prove the second assertion when $\langle U, V\rangle = -1$ you would use the alternate equation in Lemma 33:

$$||U + V||^2 = ||U||^2 + ||V||^2 + 2\langle U, V\rangle = 1 + 1 + 2(-1) = 0$$

Hence $||U + V|| = 0$ or, equivalently, $U + V = 0$ and $U = -V$.