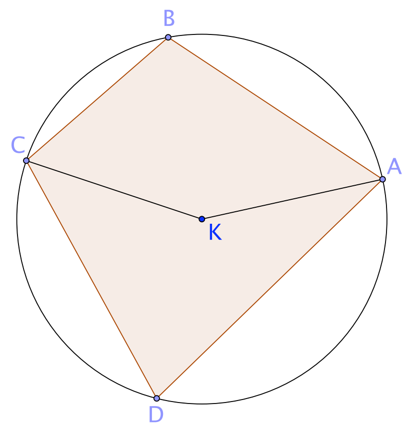
The following is a non-comprehensive list of solutions to homework problems. In some cases I may give an answer with just a few words of explanation. On other problems the stated solution may be complete. As always, feel free to ask if you are unsure of the appropriate level of details to include in your own work.

Please let me know if you spot any typos and I'll update things as soon as possible.

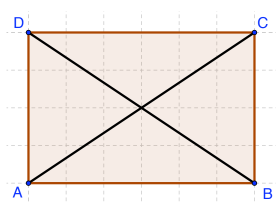
6.5.16: This problem is an application of Theorem 18. Here's our setup. Let $ABCD$ be a quadrilateral whose vertices are on a circle centered at K. Assume K is on the same side of $\overline{AC}$ as D. By relabeling points I could always ensure that's the case, unless $\overline{AC}$ goes through K, meaning it is a diameter. I'll deal with that case separately.



By the first part of Theorem 18, $|\angle AKC| = 2|\angle ADC|$. By the third part, $|\angle AKC| = 2\pi - 2|\angle ABC|$. Thus $2|\angle ADC| = 2\pi - 2|\angle ABC|$; rearrange and divide by two to get $|\angle ABC| + |\angle ADC| = \pi$. Since all four angles in this convex quadrilateral must add up to 2π, the remaining vertex angles at A and C must also add to π.

The remaining case, where $\overline{AC}$ is a diameter, is shorter than the above work. In that case the second part of Theorem 18 says the vertex angles at B and D are both $\pi/2$, in which case they certainly add to π, and the angles at A and C will add up to $2\pi - \pi = \pi$ as above.

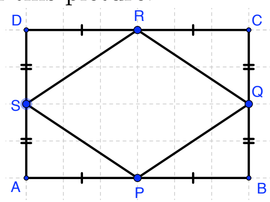
6.5.24: The fastest way to complete this problem is a "cycle" of proofs like $(i) \Rightarrow (ii) \Rightarrow (iii) \Rightarrow (i)$, which amounts to three proofs. Unfortunately in this case it seems hard to make a connection between (ii) and (iii), so it was probably faster to prove $(i) \Leftrightarrow (ii)$ and $(i) \Leftrightarrow (iii)$, a total of four proofs. I'll give **brief** outlines of the proofs below. Regardless of which condition from (i) to (iii) we're assuming, we always know that $ABCD$ in the following picture is a parallelogram, so the opposite sides are always parallel and congruent, by Proposition 3.



$(i) \Rightarrow (ii)$**:** Assume $ABCD$ is a rectangle, so all four vertex angles are congruent. Because it's also a parallelogram, $\overline{AD} \cong \overline{BC}$. Hence SAS tells us that $\triangle ABD \cong \triangle BAC$; in particular $\overline{AC} \cong \overline{BD}$, proving (ii).

$(ii) \Rightarrow (i)$**:** Now assume $\overline{AC} \cong \overline{BD}$. Using the fact that opposite sides of a parallelogram are congruent, SSS tells us that $\triangle ABD \cong \triangle BAC$; in particular $\angle A \cong \angle B$. You can use similar reasoning to show $\angle B \cong \angle C$ and $\angle C \cong \angle D$, which proves (i).

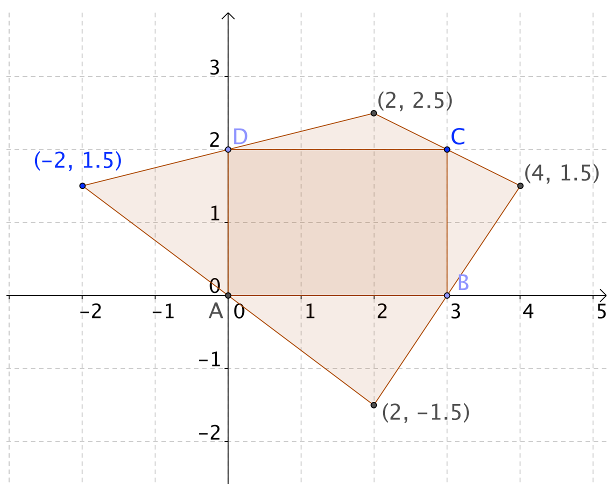
$(i) \Rightarrow (iii)$**:** Assume $ABCD$ is a rectangle, so all four vertex angles are congruent. Because it's also a parallelogram, the opposite sides are congruent. If you insert the midpoints of the sides, you can label congruent segments as shown in this picture:



Because the vertex angles of $ABCD$ are congruent, SAS tells us that the four small triangles are congruent, which means the four segments connecting P, Q, R and S in the diagram are congruent. That menas $PQRS$ is a rhombus, proving (iii).

$(iii) \Rightarrow (i)$**:** Assume $PQRS$ is a rhombus. Then all four triangles in the above picture are congruent by SSS. That means the angles at A, B, C and D are congruent, proving (i).

6.5.48: Glancing over some of the homeworks it seems most people chose kites. Here's another possibility.

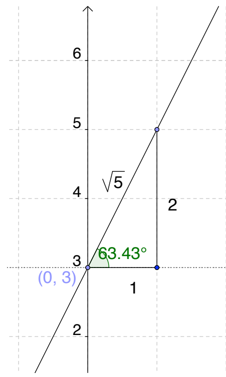


7.9.6,7: *Note that the language is a little ambiguous here about which isometry to do first, the translation or the reflection. I'll accept either answer—they turn out to be the same!—and will compute both here.*

The translation in these problems is easy to write a formula for: $\mathcal{T}(X) = X + (-3, -6)$ or, if you prefer, $\mathcal{T}(x, y) = (x-3, y-6)$. The equation for the reflection is trickier. We know $\langle (2, -1), (x, y) \rangle = -3$ is equivalent to $2x - y = -3$, or $y = 2x + 3$. So the slope of the mirror is 2, and hence the angle it forms with a horizontal line is $\theta = \arctan 2 \approx 63.4°$. The y-intercept form of the line makes it clear that $(0, 3)$ is on the line. Hence a matrix formula for the reflection across this line is:

$$\mathcal{M}(X) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \left([X] - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Here's a picture of the line:



Using the triangle in the picture, we see that

$$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$$
$$\sin 2\theta = 2\cos\theta\sin\theta = \frac{4}{5}$$

Hence our formula for the reflection becomes

$$\mathcal{M}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left([X] - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Now consider the two compositions (check the parentheses carefully to make sure you see the differences!):

$$\mathcal{T} \circ \mathcal{M}(X) = \left(\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left([X] - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} -3 \\ -6 \end{bmatrix}$$
$$\mathcal{M} \circ \mathcal{T}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\left([X] + \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right) - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

If you distribute across the parenthesis, multiply the constant vectors by the matrix, and collect terms, you'll find that these are both (!) equal to

$$\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} [X] + \begin{bmatrix} -27/5 \\ -24/5 \end{bmatrix}$$

So with these particular choices, it doesn't matter if you do the reflection first and then the translation, or vice versa; the answers to #6 and #7 are the same. (Why does it turn out that way? Hint: $(-3, -6) = -3(1, 2)$ could serve as a direction indicator for the line, so the composition in either order gives the same glide reflection!)

7.9.13: Equation (7.15), which I developed in a slightly different way in class, tells us that the formula for a central inversion (i.e. a rotation by π) about a point C is $\mathcal{C}_C(X) = -X + 2C$ or, if $C = (h, k)$,

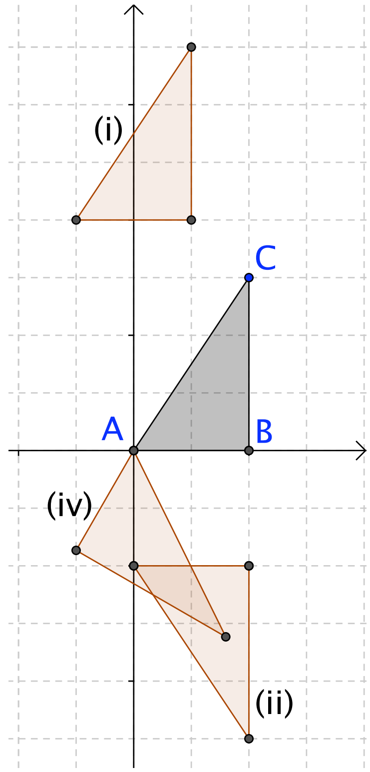
$$\mathcal{C}_C(x, y) = -(x, y) + 2(h, k) = (2h - x, 2k - y)$$

The only way $\mathcal{C}_C$ can leave (x, y) fixed (i.e. unchanged) is if

$$x = 2h - x$$
$$y = 2k - y$$

The only solution to this system is $x = h$ and $y = k$, i.e. $X = C$.

7.9.18: Here's a picture from GeoGebra showing the original $\triangle ABC$ and its images under the various isometries. Let me know if you have trouble matching the images up to the isometries, or calculating specific points, etc.


(i)
C
A
B
(iv)
(ii)