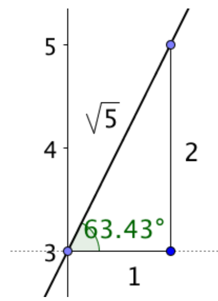
Below you'll find solutions to problems 6.6 and 6.7, which were listed on your Exam 1 Review sheet. Please let me know if you spot any typos and I'll update things as soon as possible.

6.6, 6.7: The translation in these problems is easy to write a formula for: $\mathcal{T}(X) = X + (-3, -6)$ or, if you prefer, $\mathcal{T}(x, y) = (x - 3, y - 6)$. The equation for the reflection is trickier. We know $(2, -1) \cdot (x, y) = -3$ is equivalent to $2x - y = -3$, or $y = 2x + 3$. So the slope of the mirror is 2, and hence the angle it forms with a horizontal line is $\theta = \arctan 2 \approx 63.4°$. The y-intercept form of the line makes it clear that $(0, 3)$ is on the line. Hence a matrix formula for the reflection across this line is:

$$\mathcal{M}(X) = \begin{bmatrix} \cos 2\theta & \sin 2\theta \\ \sin 2\theta & -\cos 2\theta \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Here's a picture of the line:



Using the triangle in the picture, we see that

$$\cos 2\theta = \cos\theta\cos\theta - \sin\theta\sin\theta = \frac{1}{5} - \frac{4}{5} = -\frac{3}{5}$$
$$\sin 2\theta = 2\cos\theta\sin\theta = \frac{4}{5}$$

Hence our formula for the reflection becomes

$$\mathcal{M}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

Now consider the two compositions (check the parentheses carefully to make sure you see the differences!):

$$\mathcal{T} \circ \mathcal{M}(X) = \left(\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\begin{bmatrix} X \end{bmatrix} - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} -3 \\ -6 \end{bmatrix}$$
$$\mathcal{M} \circ \mathcal{T}(X) = \begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \left(\left(\begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} -3 \\ -6 \end{bmatrix} \right) - \begin{bmatrix} 0 \\ 3 \end{bmatrix} \right) + \begin{bmatrix} 0 \\ 3 \end{bmatrix}$$

If you distribute across the parenthesis, multiply the constant vectors by the matrix, and collect terms, you'll find that these are both (!) equal to

$$\begin{bmatrix} -3/5 & 4/5 \\ 4/5 & 3/5 \end{bmatrix} \begin{bmatrix} X \end{bmatrix} + \begin{bmatrix} -27/5 \\ -24/5 \end{bmatrix}$$

So with these particular choices, it doesn't matter if you do the reflection first and then the translation, or vice versa; your answers to #6 and #7 are the same. (Why does it turn out that way? Hint: $(-3, -6) = -3(1, 2)$ could serve as a direction indicator for the line, so the composition in either order gives the same glide reflection!)